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A TREATISE

ON

ROOFS AND BRIDGES

WITH NUMEROUS EXERCISES

BY

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PREFACE.

The present treatise on Roofs and Bridges is designed as a text-book for the use of schools. The object of this work is to develop the principles and explain the methods employed in finding the forces in Roofs and Bridges, and to train the student to compute the stresses, due to the dead, live, snow, and wind loads, in the different members of any of the simple roof and bridge trusses that are in common use.

The aim has been to explain the principles clearly and concisely, to develop the different methods simply and neatly, and to present the subject in accordance with the methods used in the modern practice of roof and bridge construction.

In introducing each new truss it is at first carefully described, and the method of loading it explained. A problem is then given for this truss, and solved to determine the stresses in all the members. This problem is followed by several other similar ones, which are to be solved by the student. Nearly all of these problems were prepared especially for this work, and solved to obtain the answers. The instructor can at any time easily make up problems for his pupils without the answers.

The book consists of four chapters. Chapter I. is entirely given to Roof Trusses. Chapter II. treats only of Bridge Trusses with Uniform Loads.

Chapter III is devoted to Bridge Trusses with Unequal Distribution of the Loads. This is divided into three parts, as follows:

(1) The use of a *uniformly distributed excess load* covering one or more panels, followed by a uniform train load covering the whole span.

(2) The use of one or two *concentrated excess loads*, with a uniform train load covering the span.

(3) The use of the *actual specified locomotive wheel loads*, followed by a uniform train load.

Chapter IV treats of Miscellaneous Trusses, including the Crescent Roof Truss, the Pegram and Parabolic Bowstring Bridge Trusses, and Skew Bridges.

The stresses in this work are nearly all given in tons, the word "ton" meaning a ton of 2000 pounds. Any other unit of load and of stress might be used as well.

My best thanks are due to my friend and former pupil, Mr. George H. Blakeley, C.E., of the class of '84, now Chief Engineer of the Passaic Rolling Mill Company, for reading the manuscript and for valuable suggestions.

E. A. B.

RUTGERS COLLEGE,
NEW BRUNSWICK, N.J., October, 1898.

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ROOFS AND BRIDGES.

CHAPTER I.

ROOF TRUSSES.

Art. 1. Definitions. — A Framed Structure is a collection of pieces, either of wood or metal, or both combined, so joined together as to cause the structure to act as one rigid body. The points at which the pieces are joined together are usually called *joints.*

A Truss is a structure designed to transfer loads on it to the supports at each end, while each member of the truss is subject only to longitudinal stress, either tension or compression. The simplest of all trusses is a triangle; and all trusses, however complicated, containing no superfluous members, must be composed of an assemblage of triangles, since a triangle is the only polygon whose form cannot be changed without changing the lengths of its sides.

A Strut is a member which takes *compression.* Struts are sometimes called *posts,* or *columns.*

A Tie is a member which takes *tension.*

A Brace is a term used to denote both struts and ties.

A Counter Brace is a member which is designed to take both compression and tension. A **Counter** is a member designed to take either compression or tension; that is, for one position of the load the member may be compressed, while for another position it may be elongated.

The Upper and Lower Chords are the upper and lower members of a truss, extending from one support to the other. Each half of the upper chord of a roof truss is sometimes called the "main rafter," while the lower chord is often called the "tie rod." The upper chord is always in compression and the lower chord is always in tension. The spaces between the joints of the chords are called *panels.*

The Web Members are those which connect the joints of the two chords. They are generally alternately struts and ties.

A Roof, in common language, is the covering over a structure, the chief object of which is to protect the building from the effects of rain and snow.

A Roof Truss is a structure which supports a roof. Roof Trusses are of almost innumerable forms, and they differ greatly in the details of their construction.

The External Forces include all the exterior or applied forces, such as the weight of the structure, the weight of snow, the force of the wind, the reactions, etc., which act upon and tend to distort the structure. The external forces on the whole structure must balance each other, or else the whole structure will begin to move.

Strain is a change in the length or in the form of a body, which has been produced by the application of one or more external forces; and it is to be measured, not in tons, but in units of length, as inches or feet.

Stress is the name given to that internal force which is exerted by the material in resisting strain; and it is measured in pounds or tons the same as the external forces. It follows that, when every part of a strained member of a structure is in equilibrium, the internal stress exerted at any imaginary section through the member is equal and opposite to the straining force.

Various Kinds of Stresses. — The external forces may produce, according to circumstances, different internal forces or stresses in the various pieces of the structure. These forces or stresses and their accompanying strains may be classified as follows:

1. A direct pull or a tensile stress, producing extension or elongation.

2. A direct thrust or a compressive stress, producing compression.

3. A shearing force or a shearing stress, producing a cutting asunder.

A shearing force or shearing stress is caused by two forces acting parallel to each other, and at right angles to the axis of the piece, but in opposite directions, and in two immediately consecutive sections. The tendency is to cause two adjacent sections to slide on each other; and the term *shearing force*, or *shearing stress*, is used, because the force is similar to that in the blades of a pair of shears in the act of cutting.

Art. 2. The Dead Load. — The dead or fixed load supported by a roof truss consists of the weight of the truss itself and the weight of the roof. The weight of the truss can only be approximated. The following formula* gives approximately correct results for short spans: Let l = span in feet; b = distance between trusses in feet; and W = approximate weight of one truss in pounds. Then

$$W = \tfrac{1}{24}\, bl^2.$$

The roof includes the roof-covering, the sheeting, the rafters, and the purlins. Its total weight will vary from 5 to 30 lbs. per square foot of roof surface. The purlins are

* **Modern Framed Structures, by Johnson, Bryan, and Turneaure.**

beams running longitudinally between the trusses, and fastened to them at the upper joints, and often midway between them as well.

Art. 3. The Live Load.—The live or variable load consists of the *snow load* and the *wind load.*

The snow load varies, according to the locality, from 10 lbs. to 30 lbs. per square foot of horizontal projection. The weight of new snow varies from 5 lbs. to 12 lbs. per cubic foot, although snow which is saturated with water weighs much more. The snow load need not be considered when the inclination of the roof to the horizontal is 60° or more, as the snow would slide off.

The wind load is variable in direction and intensity, and often injurious in its effects; especially is this the case with large trusses placed at considerable intervals apart. The maximum wind pressure on a surface normal to its direction is variously estimated at from 30 lbs. to 56 lbs. per square foot. The calculation, therefore, of the stresses caused by the wind forces is often of considerable importance, and should never be left out of account in designing iron roofs of large span.

Art. 4. The Apex Loads and Reactions.—Both the dead and live loads are transferred, by means of the purlins, to the joints or apexes of the upper chords of the truss; and these loads at the apexes are called *the apex loads.* They are also called the *panel* loads.

The truss, roof, and snow loads, being vertical, and uniformly distributed over each panel, the apex loads are each equal to one half the sum of the adjacent panel loads. Thus, the load at *h,* Fig. 1, is equal to one half the panel load on *hc* plus one half the panel load on *ah.* The wind

load at h is also equal to one half the wind load on hc plus one half the wind load on ah, the load on each being normal to the surface.

If the truss be symmetrical, and the panels be of equal length, the apex loads are determined by dividing the total load by the number of panels in the upper chords. Thus in Fig. 1, the apex loads at h and c are each one fourth of the total load. At the supports, a and b, the loads are only one half those at h and c.

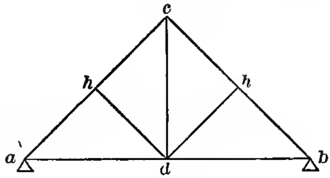


Fig. 1

Reactions. — For dead and snow loads both reactions of the supports of the truss are vertical, and each is one half of the total load, if the truss is symmetrical. For wind load the reactions depend upon the manner of supporting the truss.

Prob. 1. A roof truss, like Fig. 1, has its span 80 feet and its rise 40 feet; the distance between trusses is 12 feet, center to center. Find (1) the weight of the truss, (2) the weight of the roof, (3) the snow load, (4) the apex loads, and (5) the reactions.

For these problems, take 20 lbs. per square foot of roof surface for weight of roof, and 20 lbs. per square foot of horizontal projection for snow load.

Here $ab = 80$ feet, $dc = 40$ feet, and $ac = 56.56$ feet.

Weight of truss,	$W_t = \frac{1}{24} bl^2$	$= 3200$ lbs.
Weight of roof,	$W_r = 56.56 \times 20 \times 12 \times 2$	$= 27148.8$ lbs.
Weight of snow,	$W_s = 80 \times 12 \times 20$	$= 19200$ lbs.
Each apex load	$= \frac{1}{4} \times 49548.8$	$= 12387.2$ lbs.
Each reaction,	$R = \frac{1}{2} \times 49548.8$	$= 24774.4$ lbs.

Prob. 2. A roof truss, like Fig. 2, has its span 90 feet, its rise 30 feet, and distance between trusses 13 feet: find the

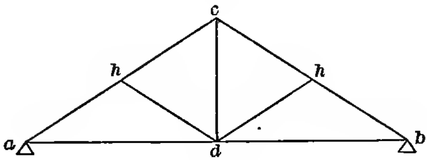


Fig. 2

total apex loads and the reactions for the weights of the truss, roof, and snow.

Ans. Apex loads = 6989 and 13977 lbs.; reaction = 27954 lbs.

Prob. 3. In the roof truss of Fig. 3, the span is 100 feet, the rise is 25 feet, and the distance between trusses 12.5 feet: find the total apex loads and the reactions for the truss, roof, and snow loads.

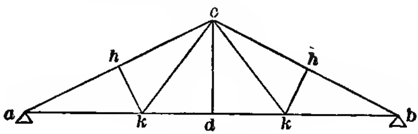


Fig. 3

Ans. Loads = 7270 and 14540 lbs.; reaction = 29079 lbs.

Art. 5. Relations between External Forces and Internal Stresses. — The external forces acting upon a truss are in equilibrium with the internal stresses in the members of the truss.

Let *mn* be a section passed through the truss, Fig. 4, cut-

ting the members whose stresses are desired, and let these stresses be replaced by equal external forces. Then it is

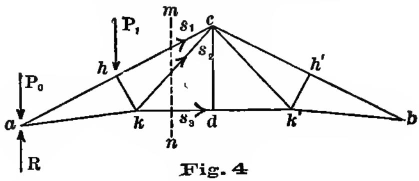


Fig. 4

clear that the equilibrium is undisturbed. Therefore we have the principle:

The internal stresses in any section hold in equilibrium the external forces acting upon either side of that section.

If we remove either portion of the truss, as the one on the right of the section, then the external forces on the remaining part of the truss, together with the internal stresses, form a system of forces in static equilibrium. And from Anal. Mechanics, Art. 61, the conditions of equilibrium for a system of forces acting in any direction in one plane on a rigid body are:

Σ horizontal components $= 0$ . . . . (1)
Σ vertical components $= 0$ . . . . (2)
Σ moments $= 0$ . . . . (3)

These three equations of condition state the relations between the internal stresses in any section, and the external forces on either side of that section. If only three of these internal stresses are unknown, they can therefore be determined.

For example, in Fig. 4, let R, P_0, P_1 be the reaction and apex loads, found as in Art. 4; let s_1, s_2, s_3 be the stresses in the members hc, kc, and kd, that are cut by the section mn, and let θ_1 and θ_2 be the angles made by s_1 and s_2 with

the vertical. Applying our three equations of equilibrium, we have:

for horizontal components, $s_1 \sin\theta_1 + s_2 \sin\theta_2 + s_3 = 0$,

for vertical components, $R - P_0 - P_1 + s_1 \cos\theta_1 + s_2 \cos\theta_2 = 0$.

In applying our third equation of condition, the center of moments may be chosen at any point, Anal. Mechanics, Art. 46. If we take it at c, the moments of s_1 and s_2 are zero, and the equation is —

$$(R - P_0)\tfrac{1}{2}ab - P_1 \times \tfrac{1}{4}ab - s_3 \times cd = 0.$$

These three equations enable us to find the unknown stresses.

NOTE 1. — In all cases, in this work, a tensile stress is denoted by the positive sign, and a compressive stress, by the negative sign.* In stating the equations, it will be convenient to represent the unknown stresses as tensile, pulling away from the section, as in Fig. 4. Then, if the numerical values of these stresses are found to be positive, it will show that they were assumed in the right direction, and are *tensile;* but if they are negative, they were assumed in the wrong direction, and are *compressive.*

Prob. 4. In the truss of Fig. 5 the span is 90 feet, the rise is 35 feet, hk is perpendicular to the rafter at its mid-point, and the loads are as shown: let it be required to find the stresses in all the members of the truss.

Representing the stresses by s_1, s_2, s_3, s_4, s_5, and s_6, we proceed to apply our three equations of equilibrium.

To find the stress s_1, pass a section cutting s_1 and s_3, separating the portion to the left, take moments about h, and regard s_1 as pulling toward the right from the section (see Note 1). Then, Σ moments about $h = 0$ gives

$$(20000 - 5000)\,22\tfrac{1}{2} - s_1 \times 17\tfrac{1}{2} = 0. \quad \therefore\ s_1 = 19287 \text{ lbs.}$$

* This convention is entirely arbitrary. Some writers denote *compression* by the positive sign, and *tension* by the negative sign.

To find s_2, pass a section cutting s_2, s_6, and s_4, take moments about c, and regard s_2 as pulling toward the right.

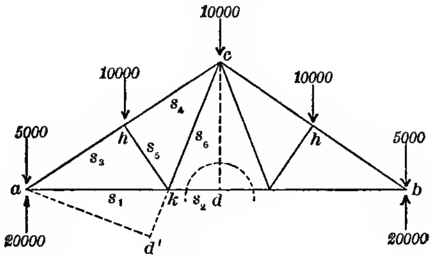


Fig. 5

Σ mom. about $c = 0$ gives

$$(20000 - 5000)\,45 - 10000 \times 22\tfrac{1}{2} - s_2 \times 35 = 0.$$

$$\therefore \; s_2 = 12857 \text{ lbs.}$$

To find s_3, pass a section cutting s_1 and s_3; we might take moments about k, but we will use our second equation instead.

Σ vert. comp. $= 0$ gives

$$20000 - 5000 + s_3 \sin cad = 0,$$

or, $$15000 + \tfrac{35}{57}\,s_3 = 0 \left(\text{since } \sin cad = \frac{cd}{ac} = \frac{35}{57}\right).$$

$$\therefore \; s_3 = -\,24428 \text{ lbs., that is, compression (see Note 1).}$$

To find s_4, pass a section cutting s_4, s_6, and s_2, take moments about k, and find the lever arms.

Σ mom. about $k = 0$ gives

$$(20000 - 5000)\,36.1 - 10000 \times 13.6 + s_4 \times 22.17 = 0$$

(since $ak = ah \sec cad = 36.1$ and $hk = ah \tan cad = 22.17$).

$$\therefore \; s_4 = -\,18290 \text{ lbs., that is, compression.}$$

To find s_5, pass the section cutting s_1, s_5, and s_4, and take moments about a. Then

$$10000 \times \frac{45}{2} + s_5 \times \frac{57}{2} = 0. \quad \therefore\ s_5 = -7895 \text{ lbs.}$$

To find s_6, pass the section cutting s_2, s_6, and s_4, and take moments about a. Then,

$$10000 \times 22.5 - s_6 \times 35 = 0 \text{ (since the lever arm of } s_6 = cd = 35\text{)},$$

$$\therefore\ s_6 = 6428 \text{ lbs.}$$

Since the truss is symmetrical and symmetrically loaded, it is evident that the stresses in all the pieces of the right half are equal to those just found in the left.

NOTE 2. — In the above solution we have called forces acting upwards and to the right, *positive*, and forces acting downwards and to the left, *negative;* also we have called moments tending to cause rotation in the direction of the hands of a clock from left to right, *positive*, and those in the opposite direction, *negative*. The opposite convention would do as well; we have only to introduce opposite forces and opposite moments with unlike signs.

Prob. 5. A roof truss like Fig. 1 has its span 40 feet, its rise 20 feet, and the apex loads 2000 lbs.: find the stresses in ad, ah, hc, and hd.

Ans. $ad = +1.5$ tons, $ah = -2.12$ tons, $hc = -1.41$ tons, $hd = -0.71$ tons.

Prob. 6. A truss like Fig. 2 has its span 60 feet, its rise 20 feet, and the panel loads 4000 lbs.: find the stresses in ad, ah, hc, and hd.

Ans. $ad = +4.5$ tons, $ah = -5.46$ tons, $hc = -3.64$ tons, $hd = -1.82$ tons.

Prob. 7. A truss like Fig. 3 has its span 80 feet, its rise 20 feet, and the panel loads 10,000 lbs.: find the stresses in all the members.

Ans. $ak = +15$ tons, $kd = +10$ tons, $ah = -16.75$ tons, $hc = -14.5$ tons, $hk = -4.5$ tons, $kc = +5$ tons, $cd = 00$ tons.

Art. 6. Methods of Calculation. — Our three equations of equilibrium, Art. 5, furnish us with two methods of calculation: *the method by resolution of forces,* and *the method of moments.* The principle of the first method is embraced in the first two equations of equilibrium; the principle of the second method is embraced in the third equation of equilibrium.

Either of these methods may be used in the solution of any given case; but in general there will be one, the employment of which in any special case will be found easier and simpler than the other. Sometimes a combination of both methods furnishes a readier solution.

REMARK. — A section may be passed through a truss in any direction, separating it into any two portions. Thus, in Fig. 5, a section may be passed around h, cutting hc, hk, and ha. Then the internal stresses s_4, s_5, s_3, and the apex load of 10,000 lbs. at h, form a system of forces in equilibrium, to which our equations are applicable.

A judicious selection of directions for the resolution of the forces often simplifies the determination of the stresses. Thus, to find s_5 in Fig. 5, if we resolve the forces into a direction perpendicular to the rafter ac, we shall obtain an equation free from the forces s_3 and s_4; whereas if the directions are taken at random, all of the forces will enter the equation. This principle is a very useful one.

Thus, we have at once in Fig. 5, calling θ the angle between the load and the rafter,

$$10000 \sin \theta + s_5 = 0. \quad \therefore \; s_5 = -10000 \cos cad = -7895 \text{ lbs.}$$

as before.

Similarly, if a section be passed around d in Fig. 5, cutting dk, dc, and db, and the vertical components be taken, the stress in dc is found at once to be zero.

In solving by the second method, the center of moments

may be taken anywhere in the plane of the forces. It is often more convenient to write three moment equations for the stresses in the three members cut, taking a new center of moments each time, than it is to use the first two equations of equilibrium; and if the center of moments be taken at the intersection of two of the pieces cut, we shall have at once the moment of the stress in the other piece, balanced by the sum of the moments of the external forces, since the moments of the stresses in the other two cut pieces are zero.

Thus, in solving Prob. 4, a section was passed cutting s_2, s_6, s_4. To find s_2 we took moments about c, the intersection of s_6 and s_4, which gave us an equation of moments containing only s_2 and known terms. To find s_4 we took moments about k, the intersection of s_2 and s_6, which gave us an equation of moments containing only s_4 and known terms. Also, to find s_6 we took moments about a, the intersection of s_2 and s_4, which gave us an equation of moments containing only s_6 and known terms.

Therefore, to find the stress in any member by the method of moments, we have the following *Rule:*

Conceive at any point of this member a section to be passed completely through the truss, cutting three members. To find the stress in this member, take the center of moments at the intersection of the other two. Then state the equation of moments between this stress and the external forces on the left of the section.

Should the section that passes completely through the truss cut more than three pieces whose stresses are unknown, if all but that piece in which the stress is required meet at a common point, the center of moments may be taken at that point.

Should the section cut but two pieces, the center of moments for either piece may be taken at any point of the other.

Prob. 8. With the dimensions and apex loads given in Prob. 5 find the stress in *cd* of Fig. 1. (See Rem.)

Ans. + 1 ton.

Prob. 9. With the dimensions and apex loads given in Prob. 6 find the stress in *cd* of Fig. 2. *Ans.* + 1.2 tons.

Prob. 10. In a truss like Fig. 4 the span is 80 feet, the rise of truss is 20 feet, the rise of tie rod is 4 feet, the panel loads are each 4 tons: find the stresses in all the members.

Ans. $ak = +18.4$ tons, $kd = +10$ tons, $ah = -20.2$ tons, $hc = -18.4$ tons, $hk = -3.6$ tons, $kc = +9$ tons, $cd = 0$ (only supports part of the tie rod).

Prob. 11. In Fig. 6 the span is 100 feet, the rise of truss

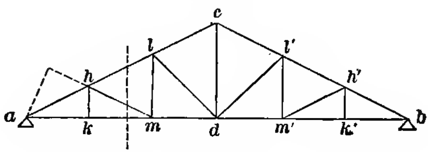


Fig. 6

is 25 feet, the apex loads are 3 tons: find the stresses in all the members.

Ans. $ah = -16.8$, $hl = -13.44$, $lc = -10.08$, $ak = +15$, $km = +15$, $md = +12$, $hk = 0$ (only supports part of the tie), $lm = +1.5$, $cd = +6$, $hm = -3.36$, $ld = -4.2$.

SUG.—The stresses in the lower chords and in all the verticals except the center one are best found by the method of moments; the

stresses in the upper chords, and also in the diagonals, may be found by the method by resolution of forces, although the stresses in the diagonals are easily found by the method of moments, taking the center at a. To find the stress in cd it is best to pass a section around d and take vertical components, as in Probs. 8 and 9.

Prob. **12.** A truss like Fig. 6 has 80 feet span, 20 feet rise, distance between trusses 12 feet, and weight of roof 20 lbs. per square foot of roof surface (see Prob. 1): find the dead load stresses in all the members, in tons.

Ans. $ah = -11.5$, $hl = -9.21$, $lc = -6.9$, $ak = +10.28 = km$, $md = +8.22$, $hk = 0$, $lm = +1.03$, $cd = +4.11$, $hm = -2.3$, $ld = -2.88$.

Art. 7. Lever Arms — Indeterminate Cases. — In determining the stresses by the method of moments the only difficulty lies in finding the *lever arms* of the various pieces. These can always be found by the use of geometry and trigonometry. Thus, in Fig. 6, the lever arms for the lower panels are evidently the perpendiculars let fall upon these panels from each opposite upper apex. The lever arms for the upper panels are the perpendiculars drawn to these panels from each opposite lower apex. The lever arm for each brace is the perpendicular to the direction of the brace drawn from the left end a of the truss, where the rafter and tie intersect. This is evident from our rule in Art. 6.

Thus, in Fig. 6, suppose a section to cut hl, hm, and km. By our rule the center of moments for km is h, the point of intersection of the other two pieces hl and hm. For hl the center is m, the intersection of hm and km. For hm the center is a, the intersection of hl and km.

For trusses in which the members have various inclina-

tions, all different, the computation of the lever arms is quite tedious. In such cases, it is sometimes advisable to make a careful drawing of the truss, and then measure the lever arms by scale. Indeed, this method can, in all cases, be used as a *check* upon the accuracy of the results obtained for the lever arm by computation.

If the section dividing the truss into two parts cuts more than three members, the stresses in which are unknown, the problem is *indeterminate*, because there are more unknown quantities to be found than there are equations of condition between them. In such cases a fourth condition is sometimes found in the symmetry of the truss and loads.

Thus, if we pass a section through *cl*, *cd*, *dl'*, and *dm'*, Fig. 6, it cuts more than three pieces. But the stress in *dl'* is equal to that in *dl*, by reason of the symmetry of the truss and loads. Even if this were not the case, it could easily be found by working toward it from the right end. Then there remain only three unknown quantities, whose values can be determined by our three equations.

The section may cut any number of members, so long as it is possible to find independently the stresses in all but three. Any truss which violates this rule is improperly framed, and has *unnecessary* or *superfluous* pieces.

REMARK. — The half of each end panel load carried by the support does not affect the truss, and need not be taken into account in finding either loads or reactions. Thus, in Fig. 5, the reaction and the half panel load acting at the support are equivalent to an upward force equal to their difference. This upward force is the reaction found by omitting the two apex loads at the supports, and is known as the *effective* or *working* reaction. Thus, instead of calling 20000 the reaction, and writing the equation

$$20000 \times 22\tfrac{1}{2} - 5000 \times 22\tfrac{1}{2} - s_1 \times 17\tfrac{1}{2} = 0,$$

as is done in the solution of Prob. 4, we call the reaction 15000 and write the equation as follows :

$$15000 \times 22\tfrac{1}{2} - s_1 \times 17\tfrac{1}{2} = 0,$$

and so for all the other moment equations.

Prob. 13. A truss like Fig. 7 has 80 feet span, 20 feet rise, distance between trusses 12 feet, and weight of roof as

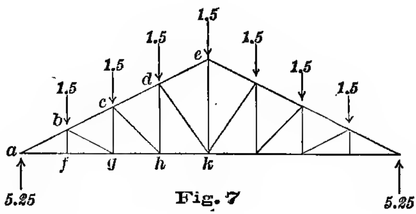


Fig. 7

before (see Prob. 1): find the dead load stresses in all the members.

Here the dead apex load

$$= \frac{12 \times \overline{80}^2}{24 \times 8} + 12 \times 20\sqrt{5^2 + 10^2} = 3083 \text{ lbs.}$$

$$= 1.54 \text{ tons, say } 1.5 \text{ tons.}$$

Effective reaction $= 1.5 \times 3.5 = 5.25$ tons (see remark).

To find $fg = af$, take moments around b.

$$5.25 \times 10 - fg \times 5 = 0. \quad \therefore\ fg = +10.5 \text{ tons.}$$

To find gh take moments around c.

$$5.25 \times 20 - 1.5 \times 10 - gh \times 10 = 0. \quad \therefore\ gh = +9 \text{ tons.}$$

To find bg pass section around g (see rem. of Art. 6).

$$10.5 + bg \times \frac{10}{11.18} - 9 = 0. \quad \therefore\ bg = -1.68 \text{ tons; and so on.}$$

Ans. $ab = -11.76$, $bc = -10.08$, $cd = -8.4$, $de = -6.72$, $af = +10.5$, $fg = +10.5$, $gh = +9$, $hk = +7.5$, $bf = 0$ (only supports part of the tie rod), $cg = +0.75$, $dh = +1.5$, $ck = +4.5$, $bg = -1.68$, $ch = -2.15$, $dk = -2.7$.

Prob. 14. In Fig. 8 the span is 60 feet, the rise of truss is 15 feet, the rise of tie rod at point 8 is 2 feet, the panel

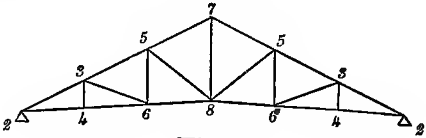


Fig. 8

loads are 1.5 tons, the struts 3–4, 5–6, 7–8 are vertical, dividing the rafter 2–7 into three equal parts: find the stresses in all the members.

Ans. Stress in 2–3 $= -9.75$, in 3–5 $= -7.8$, in 5–7 $= -5.85$, in 2–4 $= +8.73$, in 4–6 $= +8.73$, in 6–8 $= +7$, in 3–4 $= 0$ (only supports part of tie rod), in 5–6 $= +0.75$, in 7–8 $= +3.7$, in 3–6 $= -1.85$, in 5–8 $= -2.24$.

Prob. 15. In Fig. 9 the span is 90 feet, the rise of truss is 22.5 feet, the rise of tie rod is 3 feet, the struts, 3–4, 5–6,

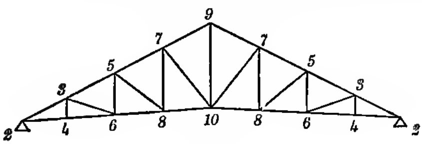


Fig. 9

7–8, and 9–10 are vertical, dividing the rafter into four equal parts; the apex loads are 2 tons: find the stresses in all the members.

Ans. Stress in 2–3 = − 18, in 3–5 = − 15.4, in 5–7 = − 12.8, in 7–9 = − 10.2, in 2–4 = + 16.1, in 4–6 = + 16.1, in 6–8 = + 13.8, in 8–10 = + 11.5, in 3–4 = 0 (this is not necessary to the stability of the truss), in 5–6 = + 1.0, in 7–8 = + 2.0, in 9–10 = + 7.2, in 3–6 = − 2.46, in 5–8 = − 2.96, in 7–10 = − 3.68.

Prob. 16. In Fig. 10 the span is 120 feet, the rise is 30 feet, the struts 3–4, 5–6, 7–8 are drawn normal to the

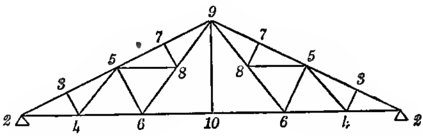


Fig. 10

rafter, dividing it into four equal parts, the apex loads are 2.5 tons: find the stresses in all the members.

Ans. Stress in 2–3 = − 19.5, 3–5 = − 18.38, 5–7 = − 17.25, 7–9 = − 16.13, 2–4 = + 17.5, 4–6 = + 15.0, 6–10 = + 10.0, 3–4 = − 2.23, 5–6 = − 4.45, 7–8 = − 2.23, 4–5 = + 2.5, 5–8 = + 2.5, 6–8 = + 5.0, 8–9 = + 7.5, 9–10 = 0 (only supports part of the tie rod, and not necessary to the stability of the truss).

It will be observed that the members 3–4 and 7–8 are symmetrical with respect to their loads, and therefore their stresses are equal, and also that 5–4 and 5–8 are symmetrical, and hence their stresses are equal.

Prob. 17. In Fig. 11 the span is 90 feet, the rise of truss is 22.5 feet, the rise of tie rod is 3 feet, the struts 3–4, 5–6, 7–8 are drawn normal to the rafter, dividing it into four equal parts; the apex loads are 2 tons: find the stresses in all the members.

Ans. Stress in 2–3 = –20.34, 3–5 = –19.44, 5–7 = –18.54, 7–9 = – 17.64, 2–4 = + 18.3, 4–6 = + 15.66, 6–10 = + 9.3, 3–4 = – 1.78, 5–6 = – 3.56, 7–8 = – 1.78, 4–5 = + 2.64, 5–8 = + 2.64, 6–8 = + 6.84, 8–9 = + 9.48, 9–10 = 0 (not necessary to stability of truss).

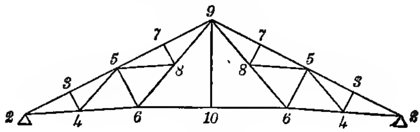


Fig. 11

Art. 8. Snow Load Stresses. — The snow load is estimated per square foot of horizontal projection (Art. 3). If the main rafters of the truss are straight — as in all of our previous problems — the snow load is uniformly distributed over the whole roof, and the apex snow loads are all equal (see Art. 4); therefore the dead load and snow load stresses are proportional to the corresponding apex loads.

Thus, in Prob. 13, the dead panel load was 3083 lbs. and the snow panel load $= 10 \times 20 \times 12 = 2400$ lbs. Therefore if we multiply each dead load stress by $\frac{2400}{3083}$ (=.778), we shall have the corresponding snow load stress.

But if the main rafters are not straight, the snow load is not uniformly distributed over the whole roof, and the apex snow loads are not all equal; in such case, the snow load stresses have to be determined independently.

Thus, with the dimensions given in Fig. 12, for 9 feet between trusses, we have the apex snow load at b and at b' each equal to one half the panel load on ab plus one half the panel load on bc

$$= 6 \times 20 \times 9 = 1080 \text{ lbs.} = 0.54 \text{ ton.}$$

The apex snow load at c equals one half the panel load on cb plus one half the panel load on cb'

$$= 8 \times 20 \times 9 = 1440 \text{ lbs.} = 0.72 \text{ ton.}$$

The stresses due to these apex snow loads may now be found in the same way as the dead load stresses were found in the preceding problems.

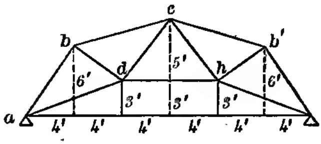


Fig. 12

It is possible for one side only of a roof to be loaded with snow. This possibility is recognized in designing roofs of very large span, such as the roof of the Jersey City train shed of the Pennsylvania R.R. In special forms of trusses such a distribution of snow load may produce a maximum stress in some members.

Problem. With the dimensions and apex snow loads above given, find the snow load stresses in all the members of Fig. 12.

Ans. $ab = -1.44$, $bc = -1.3$, $ad = +0.85$, $bd = +0.58$, $dh = +1.3$, $dc = +0.06$.

Art. 9. Wind Loads.—One of the most important questions to be dealt with in the construction of roof-trusses is the pressure of the wind; for it is evident that the stability of such a structure depends upon its power of carrying not only the weight of the truss, roof, snow, etc., but also the pressure caused by the wind. In the case of large trusses, it will often be found that, notwithstanding the great weight of the structure, the stresses produced in some of the members by a gale of wind are almost or quite

as great as those produced in the same pieces by the dead load. It appears, therefore, that in designing roofs, especially iron roofs of large span, a correct estimate of the wind loads is quite as important as a correct estimate of the dead loads.

And yet, the subject of wind forces is not well understood. It is hardly possible to define, with any precision, what degree of violence can be taken to represent the greatest wind storm that has to be provided against. In estimating the wind pressure and the resulting stresses in the members of a truss, the practice of engineers has varied greatly. Until recently, it has been considered in England sufficient to provide for a wind force of 30 to 40 lbs. per square foot of surface normal to its direction; while in this country the figures have been taken at 30 to 50 lbs. per square foot.

Taking the maximum wind pressure against a surface normal to its direction as 50 lbs. per square foot, we shall probably be on the side of safety.

The following table gives the normal wind pressure per square foot in pounds for different inclinations of the roof equivalent to a horizontal wind pressure of 50 lbs. per square foot, calculated by Hutton's formula:

Inclin.	Nor. Pres.	Inclin.	Nor. Pres.
10°	12.1	33° 30′ ($\frac{1}{3}$ span)	36.6
15°	18.0	35°	37.8
20°	22.6	40°	41.6
21° 48′ ($\frac{1}{5}$ span)	25.2	45°	43.0
25°	28.8	50°	47.6
26° 34′ ($\frac{1}{4}$ span)	30.2	55°	49.5
30°	33.0	60°	50.0

For inclinations greater than 60° the normal pressure per square foot is 50 lbs. For intermediate inclinations we can find the pressures by interpolation.

Art. 10. Wind Apex Loads and Reactions. Let Fig. 13 represent a roof truss, its span being 100 feet, and its rise 25 feet; let the distance between trusses be 12 feet, and suppose the wind to be on the left side. Then the inclination of the rafter *ae* to the horizon $= \tan^{-1}\frac{1}{2} = 26° 34'$. Hence, from our table, the normal wind pressure per square

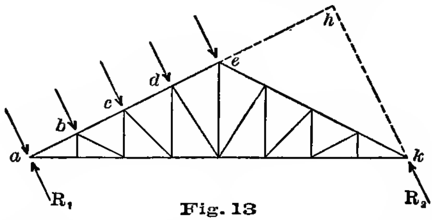


Fig. 13

foot = 30.2 lbs. The total normal wind pressure on the side of the roof *ae* is therefore

$$= 30.2 \times 12 \times \sqrt{50^2 + 25^2} = 30.2 \times 12 \times 55.9 = 20258 \text{ lbs.},$$

one fourth of which, or 5064.5 lbs., is the pressure on each panel. Hence the normal wind load at each apex *b*, *c*, *d*, is 5064.5 lbs., or say, in round numbers, 5000 lbs., or 2.5 tons, and at each apex *a*, and *e*, it is 2500 lbs., or 1.25 tons (Art. 4).

The Reactions caused by the wind pressure are inclined; the horizontal component of the wind tends to slide the entire truss off its supports. The weight of the truss and the roof are usually sufficient to cause friction enough to hold it in place. But if it is necessary, the truss should be fastened at its ends to the wall. Roof trusses of short span, and especially wooden trusses, have generally both ends fixed to the supporting walls. But large iron trusses have only one end fixed, while the other end is free, and resting upon friction rollers, so that it may move horizontally,

under changes of temperature. We have then two cases: the first, when both ends are fixed; the second, when one end only is fixed, and the other end is free to move upon rollers.

CASE I. *When both ends are fixed.*—Let Fig. 13 represent a roof truss with both ends fixed, its span being 100 feet, its rise 25 feet, and the wind apex loads 1.25, 2.5, 2.5, 2.5, and 1.25 tons, as found above. In this case, the two reactions R_1 and R_2 are parallel to the normal wind loads, and may easily be found, as follows:

Let θ be the angle between the rafter and the lower chord ak, and take moments about the left end a. We have then

$$R_2 \times ah - (2.5 \times ab + 2.5 \times ac + 2.5 \times ad + 1.25 \times ae) = 0,$$

or, we may take the resultant of all the loads, or 10 tons, acting at c,

$$\therefore\ R_2 \times 100 \cos\theta - 10 \times 27.95 = 0,$$

$$\therefore\ R_2 = 3.13 \text{ tons (since } \cos\theta = \tfrac{50}{56}).$$

The reaction R_1 may be found by subtracting R_2 from the total wind load, giving us $R_1 = 6.87$ tons. Or, we may find R_1 by taking moments about the right end k. Thus,

$$R_1 \times ah - 10\,(ah - ac) = 0,$$

from which we get $R_1 = 6.87$ tons, as before.

CASE II. *When one end is fixed and the other is free.*—(1) Suppose the right end of the truss to be free, and the wind blowing on the fixed side, as in Fig. 14. The right end of the truss is supposed to rest upon rollers, the support at a taking all the horizontal thrust due to the wind. This being the case, the reaction R_2 at the free end will be vertical, and the reaction at the fixed end will be inclined.

To find R_2 take moments about a; let the dimensions and wind loads be the same as in Fig. 13. Thus,

$$R_2 \times 100 - 10 \times 27.95 = 0. \qquad \therefore R_2 = 2.8 \text{ tons.}$$

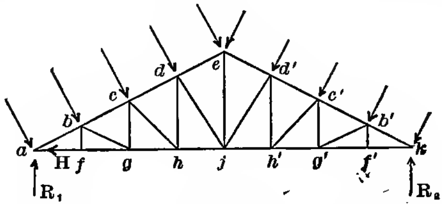


Fig. 14

Resolve the left reaction into its horizontal and vertical components, H and R_1, thus,

$$\Sigma \text{ hor. comp.} = 0$$

gives $$10 \sin\theta - H = 0. \qquad \therefore H = 4.46.$$

$$\Sigma \text{ ver. comp.} = 0$$

gives $$R_1 + 2.8 - 10\cos\theta = 0. \qquad \therefore R_1 = 6.13.$$

Or, we might find R_1 by taking moments about the right support; thus,

$$R_1 \times 100 - 10(100\cos\theta - 28) = 0. \quad \therefore R_1 = 6.13, \text{ as before.}$$

(2) Suppose the wind to blow on the free side ek of the truss, Fig. 14. The reaction R_2 is vertical, as before, and the reaction at the fixed end a may be resolved into its horizontal and vertical components, in the same way as above. Thus, we find,

$$H = 4.46, \quad R_1 = 2.8, \quad R_2 = 6.13;$$

that is, when the wind changes from one side of a roof to the other, the horizontal component H has the same value as before, but acts in the opposite direction, and the reactions R_1 and R_2 interchange their values.

Prob. 18. A truss like Fig. 14 has its span 40 feet, rise 10 feet, and the total normal wind load on the fixed side 3.2 tons: find the wind load reactions.

Ans. $R_1 = 1.96$, $R_2 = 0.90$, $H = 1.43$ tons.

Prob. 19. A truss like Fig. 3 has its span 50 feet, its rise 12.5 feet, and the distance between the trusses 8 feet: find the reactions when both ends are fixed.

Ans. $R_1 = 2.32$, $R_2 = 1.06$ tons.

Prob. 20. A truss like Fig. 4, with one end free, has its span 80 feet, its rise 20 feet, and the total normal wind load on the fixed side 5 tons: find the wind load reactions.

Ans. $R_1 = 3.07$, $R_2 = 1.4$, $H = 2.23$ tons.

Prob. 21. A truss like Fig. 4, with one end free, has its span 60 feet, rise of truss 12 feet, rise of tie rod 2 feet, and the total normal wind load on the fixed side 4.5 tons: find the wind load reactions.

Ans. $R_1 = 2.97$, $R_2 = 1.21$, $H = 1.67$ tons.

Prob. 22. A truss like Fig. 6, with one end free, has its span 90 feet, its rise 18 feet, and the total normal wind load on the fixed side 6 tons: find the wind load reactions.

Ans. $R_1 = 3.94$, $R_2 = 1.62$, $H = 2.25$ tons.

Art. 11. Wind Stresses. — (1) If the truss have *both ends* fixed, we must consider the wind blowing normally to the principal rafters on one side only (either side indifferently); and as the wind load is unsymmetrical to the roof, the wind stresses in the members on one side of the truss are different from those in the corresponding members on the other side, and hence they must be computed for every member in the truss.

(2) In trusses with *one end fixed and the other free,* we

must consider the wind blowing first on one side of the truss, and then on the other; and the stresses produced in the two cases will have to be computed.

Prob. 23. A truss like Fig. 15, with both ends fixed, has its span 50 feet, its rise 12.5 feet, and the wind loads and reactions as shown: find all the wind stresses.

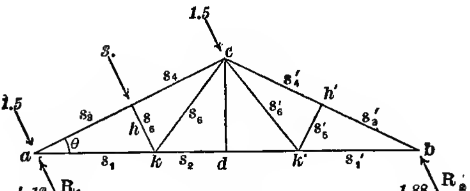


Fig. 15

We find $ac = 27.95$ feet, or, in round numbers, 28 feet; $ak = 15.68$ feet $= kc$; $hk = 7$ feet.

Representing the stresses by s_1, s_2, s_3, s_4, s_5, and s_6, and applying the principles of Arts. 5 and 6, we have for the left half of the truss:

$$2.62 \times 14 - s_1 \times 6.25 = 0, \quad \therefore \; s_1 = + 5.87 \text{ tons.}$$
$$2.62 \times 28 - 3 \times 14 - s_2 \times 12.5 = 0, \quad \therefore \; s_2 = + 2.51 \text{ tons.}$$
$$2.62 \times 14 + s_3 \times 7 = 0, \quad \therefore \; s_3 = - 5.24 \text{ tons} = s_4.$$
$$3 + s_5 = 0, \quad \therefore \; s_5 = - 3.00 \text{ tons.}$$
$$- 3 \times 14 + s_8 \times 12.5 = 0, \quad \therefore \; s_8 = + 3.36 \text{ tons.}$$

For the right half it is better to resolve the right hand reaction 1.88 into its horizontal and vertical components, thus,

$$H = 1.88 \sin \theta = 0.84, \text{ and } V = 1.88 \cos \theta = 1.68 \text{ tons,}$$

and to state the equation of each piece including the external forces on the right of the section rather than on the left. Thus,

$$1.68 \times 12.5 - .84 \times 6.25 - s_1' \times 6.25 = 0, \therefore s_1' = +2.52 \text{ tons.}$$
$$1.68 \times 25 - .84 \times 12.5 - s_2' \times 12.5 = 0, \therefore s_2' = +2.52 \text{ tons.}$$
$$1.68 \times 15.68 + s_3' \times 7 = 0, \therefore s_3' = -3.76 \text{ tons} = s_4'.$$
$$s_5' = 0, \quad s_6' = 0.$$

Prob. **24.** A truss like Fig. 15, with one end free, has its span 40 feet, its rise 10 feet, and the total normal wind load on the fixed side 3.2 tons: find all the wind stresses.

We must first find the reactions and horizontal component, as in Case II. Thus,

$$R_1 = 1.96, \quad R_2 = 0.9, \quad H = 1.43.$$

Ans. $s_1 = +3.6$, $s_2 = +1.8$, $s_3 = -2.8$, $s_4 = -2.8$, $s_5 = -1.6$, $s_6 = +1.8$, $s_1' = +1.8$, $s_3' = -2$, $s_4' = -2$, $s_5' = 0$, $s_6' = 0$.

Sug.—It will often be best to state the equation, using the external forces on the *right* of the section. Thus, to find the stress in s_2. If we use the forces on the right of the section, the equation is

$$.9 \times 20 - s_2 \times 10 = 0. \quad \therefore s_2 = 1.8.$$

But if we use the forces on the *left* of the section, the equation is

$$1.96 \times 20 + 1.43 \times 10 - 32 \times 11.18 - s_2 \times 10 = 0. \quad \therefore s_2 = 1.8.$$

Of course, to find the stresses in members near the left end, not so much is gained by using the forces on the right of the section.

Prob. **25.** A truss like Fig. 4, with one end free, has its span 60 feet, its rise 15 feet, rise of tie rod 3 feet, and the total normal wind load on the fixed side 5 tons: find the wind stresses in all the members.

Ans. Stress in $ak = +8.7$, $kd = +3.5$, $ah = -7.78$, $hc = -7.78$, $hk = -2.5$, $kc = +5.45$, $bk' = +4.36$, $bh' = -4.8$, $h'c = -4.8$, $h'k' = 0$, $ck' = +1.0$, $cd = 0$ (not necessary to stability of structure).

Prob. **26.** A truss like Fig. 14, with one end free, has its span 80 feet, its rise 20 feet, and the total normal wind load

on the fixed side 8 tons: find the wind stresses in all the members.

Ans. Stress in $af = +11.16 = fg$, $gh = +8.92$, $hj = +6.68$, $bg = -2.5$, $ch = -3.16$, $dj = -4.02$, $cg = +1.12$, $dh = +2.24$, $ej = +3.35$, $ab = -9$, $bc = -7.5$, $cd = -6.0$, $de = -4.5$, $kf' = f'g' = g'h' = h'j = +4.47$, $b'g' = c'h' = d'j = 0$, $b'f' = c'g' = d'h' = 0$, $kb' = b'c' = c'd' = d'e = -5.0$ tons.

Prob. 27. A truss like Fig. 16, with one end free, has its span 60 feet, its rise 12 feet, the struts hk and $h'k'$ normal

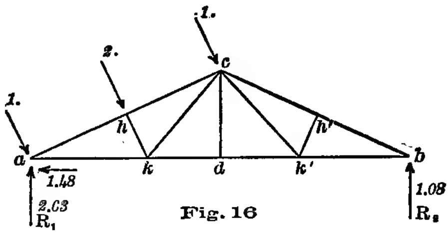


Fig. 16

to the rafters at their middle points, and the total normal wind load on the fixed side ac 4 tons: find the wind stresses in all the members.

Ans. Stress in $ak = +5.4$, $kd = +2.7$, $ah = -4.6 = hc$, $hk = -2$, $ck = +2.7$, $bk' = +2.7 = dk'$, $bh' = -2.91 = ch'$, $h'k' = 0 = ck'$.

Prob. 28. In the same truss Fig. 16, with the same dimensions as given in Prob. 27, let the same normal wind load of 4 tons blow on the free side bc: find the wind stresses in all the members.

In this problem the values of the reactions are the same as those in Prob. 27, but interchanged, and the horizontal component has the same value, but acts in the opposite direction. See Case II. of Art. 10; therefore, here $R_1 = 1.08$, $R_2 = 2.63$, $H = 1.48$.

Ans. Stress in $bk' = +3.88$, $dk' = +1.19$, $bh' = -4.58 = ch'$, $h'k' = +2$, $ck' = +2.7$, $ak = +1.22$, $dk = +1.22$, $ah = -2.91 = ch$, $hk = 0 = ck$.

Art. 12. Complete Calculation of a Roof Truss.— By comparison of the values of the stresses in Prob. 27 with those in Prob. 28, we see that the stresses are quite different, and generally greater when the wind blows on the fixed side of the roof than when it blows on the free side. When the wind blows on the fixed side it tends to "flatten" the truss; and when it blows on the free side it tends to "shut up" the truss, or "double it up."

In the complete calculation of a roof truss, we must find the stresses due to the greatest dead load, and combine them with the greatest stresses due to the live load, so as to get the greatest possible tension and compression in each member. If the dead load and live load stresses in any piece are of the same character, both compressive or both tensile, we must add them to obtain the greatest stress in the piece. But if these stresses are of opposite characters, one compression and the other tension, their difference will be the resulting stress due to the combination of live and dead loads, and if the live load stress is less in amount than the dead load stress, it will only tend, when the wind blows, to relieve the stress due to the dead load by that amount, and the dead load stress is the maximum stress in the member. But if the live load stress is greater than the dead load stress and of an opposite character, it will cause a reversal of stress and this piece will then need to be counterbraced.

It is the customary American practice to determine the greatest stresses in each member of the fixed side of the roof truss which could be caused by the wind force acting

normally to the truss on the fixed side only, and then to build the members of the two sides of the truss of the same size. Since the stresses caused by the wind blowing on the fixed side of the roof are at least as great as those caused by the wind blowing on the free side, this arrangement gives the *maximum* stresses, and is on the safe side; and for reasons of economical manufacture both sides of the truss are constructed alike.

Prob. 29. A truss like Fig. 14, with one end free, has its span 80 feet, its rise 16 feet, distance apart of trusses 16 feet, rafter divided into four equal parts, struts vertical, dead load of roof 20 lbs. per square foot of roof surface, snow load 20 lbs. per square foot of horizontal projection, normal wind load on fixed side by table of Art. 9: find the dead load, snow load, wind load, and maximum stresses in all the members.

From the given rise and span we have the length of one half of roof $ae = \sqrt{40^2 + 16^2} = 8\sqrt{29} = 43.08$ feet.

$$\text{Weight of truss} = \tfrac{1}{24}\, bl^2 \text{ (Art. 2)} = \frac{16 \times (80)^2}{24} = 4266 \text{ lbs.}$$

$$\text{Dead panel load} = \frac{4266}{8} + \frac{43.08 \times 16 \times 20}{4} = 3977 \text{ lbs.}$$

$$= 1.9885 \text{ tons, or say, 2 tons.}$$

Snow load per panel $= 10 \times 16 \times 20 = 3200$ lbs. $=1.6$ tons.

Inclin. of roof $= \tan^{-1} \cdot 4 = 21° 48'$.

Nor. pressure of wind per sq. foot (table of Art. 9) $= 25.2$ lbs.

$$\text{Nor. wind load per panel} = 25.2 \times \frac{43.08}{4} \times 16$$

$$= 4342 \text{ lbs.} = 2.171 \text{ tons, or say, 2.2 tons.}$$

From these data the dead load, snow load, and wind load stresses (wind on the fixed side only) may be computed,

and the maximum stresses found, for all the members of the truss, and tabulated, as follows:

STRESSES IN THE LOWER CHORD.

MEMBERS.	*af*	*fg*	*gh*	*hj*
Dead load stresses . .	+17.50	+17.50	+14.96	+12.44
Snow load stresses .	+14.00	+14.00	+11.97	+ 9.95
Wind load stresses . .	+14.74	+14.74	+11.77	+ 8.80
Maximum stresses . .	+46.24	+46.24	+38.70	+31.19

STRESSES IN THE UPPER CHORD.

MEMBERS.	*ab*	*bc*	*cd*	*de*
Dead load stresses . .	−18.80	−16.10	−13.40	−10.70
Snow load stresses .	−15.04	−12.88	−10.72	− 8.56
Wind load stresses . .	−12.85	−10.52	− 8.21	− 6.34
Maximum stresses . .	−46.69	−39.50	−32.33	−25.60

STRESSES IN THE WEB MEMBERS.

MEMBERS.	*cg*	*dh*	*ej*	*bg*	*ch*	*dj*
Dead load stresses	+1.00	+2.00	+ 6.00	−2.68	−3.20	− 3.90
Snow load stresses	+0.80	+1.60	+ 4.80	−2.14	−2.56	− 3.12
Wind load stresses	+1.19	+3.96	+ 3.56	−3.19	−3.78	− 5.04
Maximum stresses	+2.99	+7.56	+14.36	−8.01	−9.54	−12.06

(*bf* is not necessary to the stability of the structure.)

Here the maximum stress of each kind for each member in the windward side of the truss is found by adding the

dead load, snow load, and wind load stresses, giving the greatest total tension and the greatest total compression. Of course, the wind stresses in the members of the other half of the truss will be less than those above given for the members of the half on the windward side, and therefore, the maximum stresses in these members will be less than those in the above table, since the dead and snow load stresses in the corresponding members of the fixed and free sides of the truss are the same.

If there is no wind blowing, the maximum stresses are found by adding together the dead load and snow load stresses. If there is neither wind nor snow the dead load stresses are also the maximum stresses. If the wind blows on the free side of the truss, the maximum stresses cannot exceed those found above.

Prob. **30.** A truss like Fig. 9, with one end free, has its span 90 feet, rise of truss 18 feet, rise of tie rod 3 feet, rafter divided into four equal parts, struts vertical, dead load per panel 2 tons, snow load per panel 1.5 tons, normal wind load per panel, wind on fixed side, 2.25 tons: find the dead load, snow load, wind load, and maximum stresses in all the members of the half of the truss on the windward side.

Here the effective reaction for dead load

$$= 2 \times 3.5 = 7 \text{ tons.}$$

To find the dead load stress in 2–4 or 4–6, take moments around the point 3.

$$\therefore \text{ dead load stress in } 2\text{–}4 = \frac{7 \times 11.25}{3.75} = 21.00 \text{ tons.}$$

Similarly the dead load, snow load, and wind load stresses may be computed for all the members of the truss.

Ans. STRESSES IN THE LOWER CHORD.

MEMBERS.	2–4	4–6	6–8	8–10
Dead load stresses . .	+21.00	+21.00	+18.00	+15.00
Snow load stresses . .	+15.75	+15.75	+13.50	+11.25
Wind load stresses . .	+18.13	+18.13	+14.49	+10.84
Maximum stresses . .	+54.88	+54.88	+45.99	+37.09

STRESSES IN THE UPPER CHORD.

MEMBERS.	2–3	3–5	5–7	7–9
Dead load stresses . .	−22.60	−19.36	−16.12	−12.88
Snow load stresses . .	−16.95	−14.52	−12.09	− 9.66
Wind load stresses . .	−16.29	−13.28	−10.26	− 7.74
Maximum stresses . .	−55.84	−47.16	−38.47	−30.28

STRESSES IN THE WEB MEMBERS.

MEMBERS.	5–6	7–8	9–10	3–6	5–8	7–10
Dead load stresses	+1.00	+2.00	+ 7.60	−3.10	− 3.50	− 4.10
Snow load stresses	+0.75	+1.50	+ 5.70	−2.32	− 2.62	− 3.08
Wind load stresses	+1.21	+2.43	+ 4.54	−3.78	− 4.23	− 4.99
Maximum stresses	+2.96	+5.93	+17.84	−9.20	−10.35	−12.17

(*bf* is not necessary to the stability of the truss.)

Prob. **31.** A truss like Fig. 10, with one end free, has its span 100 feet, its rise 20 feet, the rafter divided into four equal parts by struts drawn normal to it, dead load per panel 2.5 tons, snow load per panel 1.5 tons, normal wind load per panel, wind on fixed side, 2.3 tons: find all the

stresses in all the members of the half of the truss on the windward side.

Ans. STRESSES IN THE LOWER CHORD.

MEMBERS.	2–4	4–6	6–10
Dead load stresses . .	+21.83	+18.70	+12.50
Snow load stresses . .	+13.10	+11.22	+ 7.50
Wind load stresses . .	+15.64	+12.54	+ 6.21
Maximum stresses . .	+50.57	+42.46	+26.21

STRESSES IN THE UPPER CHORD.

MEMBERS.	2–3	3–5	5–7	7–9
Dead load stresses .	−23.50	−22.57	−21.65	−20.73
Snow load stresses . .	−14.10	−13.55	−12.99	−12.44
Wind load stresses . .	−13.57	−13.57	−13.57	−13.57
Maximum stresses . .	−51.17	−49.69	−48.21	−46.74

STRESSES IN THE WEB MEMBERS.

MEMBERS.	3–4 and 7–8	5–6	4–5 and 5–8	6–8	8–9
Dead load stresses	−2.33	− 4.65	+3.10	+ 6.20	+ 9.30
Snow load stresses	−1.40	− 2.79	+1.86	+ 3.72	+ 5.58
Wind load stresses	−2.30	− 4.60	+3.11	+ 6.21	+ 9.31
Maximum stresses	−6.03	−12.04	+8.07	+16.13	+24.19

(9–10 is not necessary to the stability of the truss.)

Prob. 32. A truss like Fig. 11, with one end free, has its span 100 feet, the rise of truss 20 feet, the rise of tie rod 2.5 feet, the rafter divided into four equal parts by struts drawn normal to it, the dead, snow, and wind loads 2.5

tons, 1.5 tons, and 2.3 tons respectively, or the same as in Prob. 31: find all the stresses in all the members of the half of the truss on the windward side.

Ans. STRESSES IN THE LOWER CHORD.

MEMBERS.	2–4	4–6	6–10
Dead load stresses . .	+28.55	+24.50	+14.35
Snow load stresses . .	+17.13	+14.70	+ 8.61
Wind load stresses . .	+19.92	+17.30	+ 7.00
Maximum stresses . .	+65.60	+56.50	+29.96

STRESSES IN THE UPPER CHORD.

MEMBERS.	2–3	3–5	5–7	7–9
Dead load stresses . .	−30.60	−19.68	−28.75	−27.83
Snow load stresses . .	−18.36	−17.81	−17.25	−16.70
Wind load stresses . .	−18.12	−18.12	−18.12	−18.12
Maximum stresses . .	−67.08	−55.61	−64.12	−62.65

STRESSES IN THE WEB MEMBERS.

MEMBERS.	3–4 and 7–8	5–6	4–5 and 5–8	6–8	8–9
Dead load stresses	−2.33	− 4.65	+ 4.00	+10.65	+14.65
Snow load stresses	−1.40	− 2.79	+ 2.40	+ 6.39	+ 8.79
Wind load stresses	−2.30	− 4.60	+ 4.02	+ 9.15	+13.16
Maximum stresses	−6.03	−12.04	+10.42	+26.19	+36.60

(9–10 is not necessary to the stability of the truss.)

Prob. 33. A truss like Fig. 11, with one end free, has its span 120 feet, rise of truss 20 feet, rise of tie rod 3 feet, rafter divided into four equal parts by struts drawn normal

to it, distance between trusses 20 feet, dead load of roof 15 lbs. per square foot of roof surface, snow load 20 lbs. per square foot of horizontal projection, normal wind load on fixed side by table of Art. 9: find all the stresses in all the members in the half of the truss on the fixed side.

Ans. Dead load per panel = 3.1 tons; snow load per panel = 3 tons; wind load per panel = 3.3 tons.

STRESSES IN THE LOWER CHORD.

MEMBERS.	2–4	4–6	6–10
Dead load stresses	+ 45.26	+ 38.91	+22.17
Snow load stresses	+ 43.80	+ 37.65	+21.45
Wind load stresses	+ 37.35	+ 29.92	+12.56
Maximum stresses	+126.41	+106.48	+56.18

STRESSES IN THE UPPER CHORD.

MEMBERS.	2–3	3–5	5–7	7–9
Dead load stresses	– 47.52	– 46.56	– 45.60	– 44.64
Snow load stresses	– 45.99	– 45.06	– 44.13	– 43.20
Wind load stresses	– 35.31	– 35.31	– 35.31	– 35.31
Maximum stresses	–128.82	–126.93	–125.04	–123.15

STRESSES IN THE WEB MEMBERS.

MEMBERS.	3–4 and 7–8	5–6	4–5 and 5–8	6–8	8–9
Dead load stresses	–2.95	– 5.89	+ 6.36	+17.36	+23.72
Snow load stresses	–2.85	– 5.70	+ 6.15	+16.80	+22.95
Wind load stresses	–3.35	– 6.70	+ 7.44	+17.76	+25.19
Maximum stresses	–9.15	–18.29	+19.95	+51.92	+71.86

When the members of a truss have many different inclinations, as, for example, in the Crescent Truss and Arch Truss, the calculation of the stresses by the method of moments and the method by resolution of forces (Art. 6), becomes very tedious, owing to the difficulty in finding the lever arms, or the sines and cosines. In practice the graphic method is often used for determining the stresses in such trusses; but if the analytic method is preferred to the graphic, the truss should be drawn to a large scale, and all the lever arms and sines and cosines be scaled from the diagram.

The present chapter is but a brief treatise on roof trusses. The student who desires to continue the subject further is referred to more extended works, such as "Strains in Framed Structures," by Du Bois, and "Theory and Practice of Modern Framed Structures," by Johnson, Bryan, and Turneaure.

CHAPTER II.

BRIDGE TRUSSES WITH UNIFORM LOADS.

Art. 13. Definitions. — **A Bridge** is a structure for carrying moving loads over a body of water, or over a depression in the earth. For railroad and highway accommodations, it connects two roads so as to form a continuous road. A *framed bridge*, or *trussed bridge* is composed of two or more trusses, which lie in vertical planes, parallel to the line of the road. A bridge truss, like a roof truss, consists of the upper and lower chords and the web members. The upper chord of a simple* bridge truss is in compression, and the lower is in tension, while the web members are some in compression and some in tension. The trusses are united by *lateral bracing*, attached to the panel points of either the upper or lower chords, or both. The object of this lateral bracing is to support the trusses sideways, and stiffen the structure against the action of the wind.

The Floor, or **Floor System**, of a bridge consists of the *floor beams*, *stringers*, and *flooring*. The *floor beams* run at right angles with the chords, and are connected to them at the panel points. The *stringers* frame into or rest upon the floor beams, and are parallel to the chords, and support the flooring of a highway bridge, or the cross ties and rails of a railroad bridge.

* This is true only for simple trusses, and not for cantilever trusses, continuous trusses, or trusses of draw spans.

A Through Bridge is one in which the roadway is supported by the bottom chords, with lateral bracing overhead between the top chords.

A Deck-bridge is one in which the roadway is supported by the top chords; the trusses are usually placed nearer to each other than on through bridges, the roadway extending over them.

When a through bridge is not of sufficient height to allow of the upper lateral bracing, the trusses are called *Pony Trusses.* Such trusses are necessarily short; they are "stayed" by bracing connected with the floor system.

Art. 14. Different Forms of Trusses.—The King-post Truss is a term applied to a truss in which there is a central vertical tie and two braces resting against it, as in Fig. 17. It was formerly built for country highway bridges

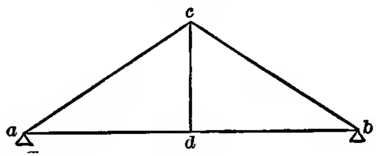


Fig. 17

of short span. The term "king-post" is an old one, and came into use when the central piece was made of wood, and resembled a *post,* although its office was that of a *tie;* this tie is now usually a wrought iron rod. The load at the center *d* is carried by the tie up to the apex *c,* and then by the two inclined braces *ca* and *cb* down to the abutments, *a* and *b.*

The Queen-post Truss is a truss in which there are two vertical *ties* against which rest two braces, as in Fig. 18.

This is also an old term and has become familiar with long use. It is sometimes called a *trapezoidal truss.* The panel

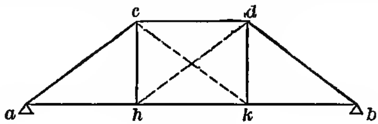


Fig. 18

loads at *h* and *k* are carried up the ties to *c* and *d*, and then by the two inclined struts *ca* and *db* down to the abutments, *a* and *b*.

The Howe Truss, Fig. 19, has its vertical members in tension and the inclined ones in compression; the diagonal counter struts are in broken lines. The chords and diagonal

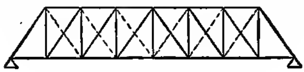


Fig. 19

web members are of wood, and the vertical ties of iron. This truss was patented in the United States in 1840 by William Howe. It has proved the most useful style of bridge truss ever devised for use in a new and timbered country. It is still very largely used where timber is cheap, for both highway and railway bridges.

The Pratt Truss, Fig. 20, has its vertical members in compression and the inclined ones in tension. All the

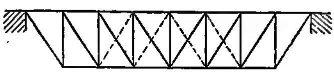


Fig. 20

members of this truss are of iron or steel; though it was formerly built all of wood, except the diagonal ties, which

were of wrought iron. The Pratt truss is a favorite type, and is used more than any other kind. Figure 20 shows a *deck*-bridge. The deck or through Pratt truss is the standard form of truss for both highway and railway bridges of moderate spans, though it is not generally used for railway bridges in which the span is much less than 100 feet.

The Warren Truss, or **Warren Girder**, has all its web members inclined at equal angles, some of them being in tension and some in compression. This truss is an example of the pure triangular type. Its web members consist always of *equilateral triangles.* When the triangles are not equilateral, the truss is simply a "*triangular* truss."

Fig. 21

Figure 21 shows a Warren truss as a *deck* truss. The Warren truss is generally built all of iron or steel, and is used for comparatively short spans; it is of more frequent occurrence in England than in this country.

The Double Triangular, or **Double Warren Truss** (or **Girder**), Fig. 22, has each panel braced with two diagonals intersecting each other, and forming a single lattice.

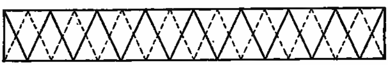


Fig. 22

Other forms of trusses will be explained as we proceed. All these forms of trusses, and bridge trusses generally, may be arranged so as to be used either for deck-bridges or through bridges.

Art. 15. The Dead Load, or permanent load, consists of the entire weight of the bridge. It includes the weight of the trusses, the lateral bracing, and the floor system. In *highway bridges,* the floor system consists of the floor beams, which are supported by the chords at the panel points, the stringers, which are supported by the floor beams, and the planks, which are supported by the stringers. In *railway bridges* the stringers support the cross ties, rails, guard-rails, spikes, etc. The dead load depends upon the length of the bridge, its width, its style, and upon the live loads it is intended to carry. For highway bridges with plank floors the total dead load per foot may be found approximately by the following formula:

$$w = 150 + cbl + 4\,bt,$$

where $w =$ weight in pounds per linear foot of bridge.

$l =$ length of bridge in feet,

$b =$ width of roadway in feet,

$t =$ thickness of planking in inches,

$c = \frac{1}{5}$ for heavy city bridges,

$= \frac{1}{6}$ for ordinary city or suburban bridges,

$= \frac{1}{7}$ for light country bridges.

For single track railroad bridges the dead load per linear foot is given very closely by the following formulæ (taken from "Modern Framed Structures," by Johnson, Bryan, and Turneaure, p. 44).

For deck-plate girders,

$$w = 9\,l + 520 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

For lattice girders,

$$w = 7\,l + 600 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

For through pin-connected bridges,

$$w = 5l + 750 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

For Howe trusses, $w = 6.5l + 675 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$

where l is the span in feet, and w the dead load in pounds per linear foot. These four formulæ give dead loads of iron railway bridges, including an allowance of 400 lbs. per foot for the weight of track material, for bridges designed to carry 100-ton locomotives. For lighter locomotives the dead load would be less, and for heavier locomotives it would be more. For double track bridges add 90 per cent to the above values;* for the load on each truss take one half the above values.

Prob. 34. What is the weight of a Warren truss bridge 105 feet long?

Here we may find the dead load from formula (2), as follows:

Dead load per linear foot

$$= w = 7l + 600 = 1335 \text{ lbs.}$$

$\therefore$ weight of bridge $= 1335 \times 105 = 140{,}175$ lbs. and weight of bridge to be carried by one truss

$$= 70087.5 \text{ lbs.}$$

Prob. 35. What is the weight of a through pin-connected bridge of 100 feet span?

Ans. 125,000 lbs.

Prob. 36. What is the weight of a Howe truss bridge of 144 feet span?

Ans. 231,984 lbs.; or 115,992 lbs. per truss.

* For double track bridges the weight of the *metal work* is about 90% greater than for a single track; but the weight of the *track material* is just double that for a single track bridge.

Art. 16. The Live Load, or moving load, is that which moves over the bridge, and consists of wagons and foot passengers on highway bridges, and trains on railway bridges.

For *highway bridges,* the live load is usually taken as a uniform load of from 50 to 100 lbs. per square foot of roadway, or the heaviest concentrated load, due to a road-roller, which is likely to come upon the bridge. The trusses of highway bridges are usually found to receive the greatest stresses from a densely packed crowd of people, while the floor system usually receives the greatest stresses from the concentrated load. The following are the live loads specified by Waddell, per square foot of floor:

Span of Bridge.	City and Suburban Bridges.	Country Bridges.
0 to 50 feet.	100 lbs.	90 lbs.
50 to 150 feet.	90 lbs.	80 lbs.
150 to 200 feet.	80 lbs.	70 lbs.
200 to 300 feet.	70 lbs.	60 lbs.
300 to 400 feet.	60 lbs.	50 lbs.

The live load per panel of a highway bridge may then be found by multiplying the width of roadway including the sidewalks by the product of the load per square foot with the panel length.

For railway bridges the live load is often specified as the weight of two of the heaviest locomotives which we think will ever pass over the bridge, and followed by a uniform load due to the heaviest possible train.

In English, and sometimes in American practice, it is considered sufficiently accurate to use the corresponding *uniformly distributed* load which causes the same stresses in the chords as those which result from the above concentrated locomotive loads.

The following equivalent uniformly distributed live loads per linear foot of track, produce approximately the same stresses as the above concentrated loads.

Span, 10, 20, 30, 40, 50, 100, 200, 300 feet.

Load, 10,000, 6600, 5500, 4900, 4600, 4000, 3700, 3500 lbs.

This loading is very nearly that used by the Pennsylvania Railroad. The Erie Railroad uses a live load one fifth greater; and the Lehigh Valley Railroad a live load one third greater.

The live load is taken greater for short spans than for long ones, because if the span is short one or two locomotives may cover the whole bridge, while if the span is long, the whole bridge would not often be loaded with more than a train drawn by one or two locomotives. The calculations of the stresses are made in precisely the same way for railway as for highway bridges, so long as the live load is uniform.

In the use of equivalent uniform loads, the following precautions are to be observed: In obtaining the stresses in the chords and the main web members, the equivalent load corresponding to the length of the *truss* is to be used. But in such members as receive a maximum stress from a single panel load, another equivalent load must be used. Thus, in the "hip verticals" of a Pratt truss, Fig. 30, or in the "vertical suspenders" of a Warren truss, Fig. 35, the maximum stress is obtained by using the equivalent load corresponding to a *span of two panel lengths.*

Prob. 37. A bridge for a city has its roadway 20 feet wide in the clear, and also two sidewalks, each 6 feet wide in the clear. The span is 200 feet and there are 10 panels. 'Find the live panel load per truss.

Ans. 12.8 tons.

Prob. 38. A country bridge, 50 feet long, has its roadway 16 feet wide in the clear, and a sidewalk 8 feet wide in the clear. There are 5 panels. Find the live panel load per truss.

Ans. 5.4 tons.

Art. 17. Shear — Shearing Stress. — Let Fig. 23 represent a beam fixed horizontally at one end and sustaining a load P at the other end. Imagine the beam divided into vertical slices or transverse sections of small thickness. The weight P tends to separate or shear the section or slice on which it immediately rests from the adjoining one. The lateral connection of the sections prevents this separation, and the second section or slice is drawn by a vertical force equal to the weight P which tends to slide or shear it from the third section, and so on. Thus, a vertical force equal to the weight P is transmitted from section to section throughout the length of the beam to the point of support. This vertical force is called the "*shearing force,*" or "*shear*"; and the equal and opposite internal force or stress in the section that balances it is called the "*shearing stress.*"

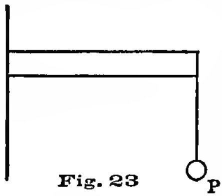


Fig. 23

The shear then at any section is that force which tends to make that section slide upon the one immediately following.

The vertical shear at any section is the total vertical force at that section, and it is equal to *the sum of the vertical components of all the external forces acting upon the beam on either side of the section.*

Thus, let Fig. 24 represent a beam l feet long, resting horizontally on supports at its extremities; and let the

beam have a uniform load of w lbs. per foot. Consider a section ab at any distance x from the left end. Then, the reaction at each support is $\frac{1}{2}\ wl$; the shear in the section ab is the weight that is between that section and the center

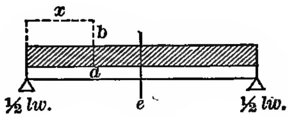


Fig. 24

of the beam, or $(\frac{1}{2}\,l - x)w$, since this is the total vertical force at ab, which is the shear by definition; but this is the same as the algebraic sum of the vertical components on the left side of ab.

For the algebraic sum of the vertical components on the left of the section $ab = \frac{1}{2}\,lw - xw$, or $(\frac{1}{2}\,l - x)w$.

Art. 18. Web Stresses due to Dead Loads — Horizontal Chords. — Let $ABCD$ be a truss with horizontal

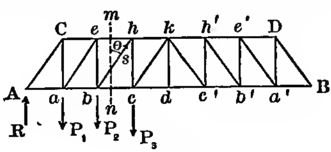


Fig. 25

chords* AB, CD, and let a section mn be drawn in any panel bc cutting the diagonal bh and the two chords, and let θ be the angle which the diagonal makes with the vertical. Then (Art. 5) the stresses in these three members must be in equilibrium with the external forces on the left of the section; and therefore the algebraic sum of the

* Called also flanges.

vertical components of these forces equals zero. Hence, calling S the stress in the diagonal bh, we have

$$R - P_1 - P_2 + S\cos\theta = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

But $R - P_1 - P_2$ is the vertical shear for the given section mn (Art. 17).

Hence (1) becomes

$$\text{Shear} + S\cos\theta = 0;$$

$$\therefore\ S = -\text{ shear} \sec\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Therefore, for horizontal chords and vertical loads, *the stress in any web member is equal to the vertical shear multiplied by the secant of the angle which the member makes with the vertical.*

Since the shear in the section cutting hc is the same as that in the section mn, no load being at h, therefore the stress in Ch is equal to the shear in ch, that is, equal to the shear in the panel bc. Since there are no loads between the joints b and c, the shear is constant throughout the panel bc; and we usually speak of it as the shear in the panel bc.

It will be observed from (2) that, for the diagonal bh, the stress is *negative*, or *compressive*, provided that the shear is *positive;* but for the dead load the shear is always positive in sections left of the middle of the truss. Hence the stress in bh is compressive, and so for all the other diagonals in the left half of the truss, since they are all inclined in the same direction, that is, downwards toward the left. Conversely, for members inclining downwards toward the right, in the left half of the truss, the stresses are *positive* or *tensile.*

Prob. 39. A through Howe truss, like Fig. 25, has 8 panels, each 15 feet long, and 20 feet deep: find all the

web stresses due to a dead load of 450 lbs. per linear foot per truss.

$$\text{Panel load} = \frac{450 \times 15}{2000} = \frac{27}{8} = 3.38 \text{ tons.}$$

$$\text{Reaction} = \frac{27}{8} \times \frac{7}{2} = \frac{189}{16} = 11.8 \text{ tons,}$$

which is also the shear in panel *Aa*. The shear in each of the other panels is found by subtracting from the reaction the loads on the left of the panel.

$$\sec\theta = \frac{\sqrt{15^2 + 20^2}}{20} = \frac{5}{4} = 1.25.$$

The secant for the verticals = 1.

We have then the following stresses for the *diagonals:*

$$\begin{aligned}
&\text{Stress in } AC = -\ 11.8 \times 1.25 &&= -14.8 \text{ tons.}\\
&\text{Stress in } ae = -(11.8 - 3.38)1.25 &&= -10.5 \text{ tons.}\\
&\text{Stress in } bh = -(11.8 - 6.76)1.25 &&= -\ 6.3 \text{ tons.}\\
&\text{Stress in } ck = -(11.8 - 10.14)1.25 &&= -\ 2.1 \text{ tons.}
\end{aligned}$$

For the verticals:

Stress in $aC = +11.8$ tons. Stress in $be = +8.4$ tons.
Stress in $ch = +5.1$ tons. Stress in $dk = +3.4$ tons.

This last value is found by passing a section around d cutting dc, dk, dc' (see Rem. of Art. 5), and taking vertical components.

Prob. 40. A deck Pratt truss, like Fig. 20, has 12 panels, each 6 feet long, and 6 feet deep: find all the web stresses due to a dead load of 500 lbs. per linear foot per truss.

Ans. Stresses in verticals

$= -8.25,\ -6.75,\ -5.25,\ -3.75,\ -2.25,\ -1.50$ tons.

Stresses in diagonals

$= +11.63,\ +9.51,\ +7.40,\ +5.28,\ +3.17,\ +1.05$ tons.

Prob. **41.** A through Warren truss, Fig. 26, has 10 panels, each 10 feet long, its web members all forming equilateral

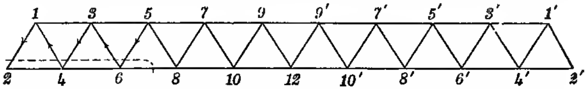


Fig. 26

triangles (Art. 14): find the stresses in all the web members due to a dead load of 400 lbs. per linear foot per truss.

Ans. Stresses in 1–2, 3–4, 5–6, 7–8, 9–10 are -10.38, -8.08, -5.76, -3.46, -1.14 tons.

Stresses in 1–4, 3–6, 5–8, 7–10, 9–12, are $+10.38$, $+8.08$, $+5.76$, $+3.46$, $+1.14$ tons; that is, the signs alternate in the web members.

Art. 19. Chord Stresses due to Dead Loads — Horizontal Flanges.

(1) *To find the chord stresses by the method of moments.*

Pass a section cutting the chord member whose stress is required, a web member, and the other chord, and take the center of moments at the intersection of the web member and the other chord. Then, supposing the right part of the truss removed, state the equation of moments between the unknown stress and the exterior forces on the left of the section. For stresses in the upper chord members the centers of moments are at the lower panel points; and for stresses in the lower chord members the centers are at the upper chord points.

Thus, in Prob. 41, each panel load is 2 tons, and each reaction is therefore 9 tons; the depth of the truss is $10 \sin 60° = 8.66$ feet. Hence for the lower chord stresses, we have:

$$9 \times 5 - \text{stress in } 2\text{–}4 \times 8.66 = 0;$$
$$\therefore \text{ Stress in } 2\text{–}4 = +5.20 \text{ tons.}$$
$$9 \times 15 - 2 \times 5 - \text{stress in } 4\text{–}6 \times 8.66 = 0;$$
$$\therefore \text{ Stress in } 4\text{–}6 = +14.43 \text{ tons.}$$
$$9 \times 25 - 2(15 + 5) - \text{stress in } 6\text{–}8 \times 8.66 = 0;$$
$$\therefore \text{ Stress in } 6\text{–}8 = +21.36 \text{ tons.}$$
$$9 \times 35 - 2 \times 45 - \text{stress in } 8\text{–}10 \times 8.66 = 0;$$
$$\therefore \text{ Stress in } 8\text{–}10 = +25.98 \text{ tons.}$$
$$9 \times 45 - 2 \times 80 - \text{stress in } 10\text{–}12 \times 8.66 = 0;$$
$$\therefore \text{ Stress in } 10\text{–}12 = +28.28 \text{ tons};$$

and for the upper chord stresses, we have:

$$9 \times 10 - \text{stress in } 1\text{–}3 \times 8.66 = 0;$$
$$\therefore \text{ Stress in } 1\text{–}3 = -10.38 \text{ tons.}$$
$$9 \times 20 - 2 \times 10 - \text{stress in } 3\text{–}5 \times 8.66 = 0;$$
$$\therefore \text{ Stress in } 3\text{–}5 = -18.48 \text{ tons.}$$
$$9 \times 30 - 2(20 + 10) - \text{stress in } 5\text{–}7 \times 8.66 = 0;$$
$$\therefore \text{ Stress in } 5\text{–}7 = -24.24 \text{ tons.}$$
$$9 \times 40 - 2 \times 60 - \text{stress in } 7\text{–}9 \times 8.66 = 0;$$
$$\therefore \text{ Stress in } 7\text{–}9 = -27.70 \text{ tons.}$$
$$9 \times 50 - 2 \times 100 - \text{stress in } 9\text{–}9 \times 8.66 = 0;$$
$$\therefore \text{ Stress in } 9\text{–}9 = -28.86 \text{ tons.}$$

(2) *By the method of chord increments.*

Let it be required to find the stress in the chord member 6–8, Fig. 26. Denote the vertical shears in the web members 2–1, 1–4, 4–3, etc., by v_1, v_2, v_3, etc., and the angles these members make with the vertical by θ_1, θ_2, θ_3, etc. Now pass a curved section cutting the chord member 6–8 and all the web members on the left. Then from the first

condition of equilibrium the sum of the horizontal components is zero. But the horizontal component of the stress in any web member is equal to the vertical shear in that member multiplied by the tangent of its angle with the vertical. Hence we have:

Stress in 6–8

$$= v_1 \tan \theta_1 + v_2 \tan \theta_2 + v_3 \tan \theta_3 + v_4 \tan \theta_4 + v_5 \tan \theta_5.$$

Therefore, to find the stress in any chord member, pass a curved section cutting the member and all the web members on the left, multiply the vertical shear in each web member by the tangent of its angle with the vertical, and take the sum of the products.

From an inspection of Fig. 26 it is seen that the stress in any chord member, as 4–6 for example, is greater than the stress in the immediately preceding chord member, 2–4, by the sum of the horizontal components of the stresses in the web members, 1–4 and 4–3, intersecting at the panel point between these two members; That is, *the increment of chord stress at any panel point is equal to the sum of the horizontal components of the web stresses intersecting at that point; or, equal to the sum of the products of the vertical shears of these web members by the tangents of their respective angles with the vertical.*

It is well to test the stress found by this method by the method of moments, as the results obtained by the two methods should agree; and this affords a check on the work.

The method by chord increments applies only to *horizontal* chords, while the method by moments applies to trusses of *any form*, that is, when the chords are *not* horizontal as well as when they are.

Prob. **42.** Let it be required to find the chord stresses in Prob. 41 by the method of chord increments.

Here each panel load is 2 tons, and all the web members are inclined at an angle of 30°;

$$\therefore \tan \theta_1 = \tan \theta_2 = \text{etc.} = \tan 30° = .5773.$$

Hence for the lower chord stresses we have:

Stress in 2–4 $= 9 \times .5773 = +5.20.$
Stress in 4–6 $= (9 + 9 + 7) \times .5773 = +14.43.$
Stress in 6–8 $= (9+9+7+7+5) \times .5773 = +21.36.$
Stress in 8–10 $= (32+5+5+3) \times .5773 = +25.98.$
Stress in 10–12 $= (42+3+3+1) \times .5773 = +28.29;$

and for the upper chord stresses we have:

Stress in 1–3 $= -(9 + 9) \times .5773 = -10.39.$
Stress in 3–5 $= -(18+2\times 7) \times .5773 = -18.47.$
Stress in 5–7 $= -(32+2\times 5) \times .5773 = -24.25.$
Stress in 7–9 $= -(42+2\times 3) \times .5773 = -27.71.$
Stress in 9–9 $= -(48+2\times 1) \times .5773 = -28.86.$

Prob. **43.** Find all the chord stresses in the deck Pratt truss of Prob. 40.

Ans. Stresses in lower chords
$= +8.25, +15.0, +20.25, +24.0, +26.25$ tons.
Stresses in upper chords
$= -8.25, -15, -20.75, -24, -26.25, -27.0$ tons.

Prob. **44.** A through Howe truss, like Fig. 25, has 12 panels, each 10 feet long, and 10 feet deep: find the stresses in all the members due to a dead load of 400 lbs. per linear foot per truss.

Ans. Lower chords = 11.0, 20.0, 27.0, 32.0, 35.0, 36.0 tons.
Verticals = 11.0, 9.0, 7.0, 5.0, 3.0, 2.0 tons.
Diagonals = 15.5, 12.68, 9.86, 7.04, 4.22, 1.4 tons.

Of course the upper chord stresses can be written directly from those of the lower chord by (1) of Art. 5.

Art. 20. Position of Uniform Live Load Causing Maximum Chord Stresses.—Let Fig. 27 be a truss sup-

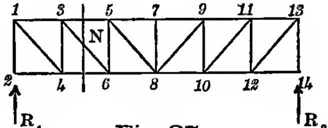


Fig. 27

ported at the ends. Then to find the stress in any chord member, we pass a section cutting that member, a web member, and the other chord, and take the center of moments at the intersection of the web member and the other chord; and since the lever arm for the chord is constant, the stress in any chord member will be greatest when the live load is so arranged as to give the greatest bending moment.

Now suppose we have a uniformly distributed moving load coming on the truss from the right, till it produces the left abutment reaction R_1; then any increase in the load on the right of the section N will affect the forces on the left only by increasing the reaction R_1, and consequently the bending moment. Hence, as R_1 increases with every load added to the right of N, the bending moment increases, and therefore the chord stress also increases. Also, suppose we have a uniformly distributed moving load covering the truss on the left of the section producing the right abutment reaction R_2; then any increase in the load on the left of the section N will affect the forces on the right only by increasing the reaction R_2, and consequently the bending moment. Hence every load, whether on the right or left of the section, increases the bending moment, and therefore the chord stress.

Therefore, for a uniform load, the maximum bending moment at any point, and consequently the maximum chord stress in any member, occurs when the live load covers the whole length of the truss.

To determine the chord stresses then due to a uniform live load, we have only to suppose the live load to cover the whole truss, just as the dead load does, and compute the chord stresses in exactly the same way as we compute the dead load stresses.

Prob. 45. A through Warren truss, like Fig. 26, has 8 panels, each 8 feet long: find the stresses in all the chord members due to a live load of 1000 lbs. per linear foot per truss.

Ans. Upper chord stresses = 16.16, 27.72, 34.64, 36.96 tons.
Lower chord stresses = 8.08, 21.92, 31.20, 35.80 tons.

Prob. 46. A deck Pratt truss has 11 panels, each 11 feet long, and 11 feet deep: find all the chord stresses due to a live load of 800 lbs. per linear foot per truss.

Ans. Upper chord stresses
= 22.0, 39.6, 52.8, 61.6, 66.0, 66.0 tons.

Art. 21. Maximum Stresses in the Chords.—According to the principles of the preceding Article the stresses in the chords will be greatest when both dead and live loads cover the whole truss. We have then only to determine the stress in each chord member due to the dead and live loads, as in Arts. 19 and 20, and take their sum; or, we may add together at first the dead and live panel loads, and determine the maximum stress in each chord member directly. This method is the simplest and shortest; but it is not the one which in practice is generally employed.

All modern specifications require a separation of dead and live load stresses. One type of specification permits twice the allowable stress per square inch of metal for dead load that it permits for live load; thus 10,000 lbs. per square inch for live load stresses and 20,000 lbs. per square inch for dead load stresses. So that if a tension member has a live load stress of 100,000 lbs. and a dead load stress of 50,000 lbs., the sectional area is determined as follows:

$$\frac{100000}{10000} = 10 \text{ sq. in.} \qquad \frac{50000}{20000} = 2\tfrac{1}{2} \text{ sq. in.}$$

That is, $12\frac{1}{2}$ square inches are required for the sectional area of the member.

Another type of specification requires the sectional area to be determined by a consideration of the minimum and maximum stresses. Thus, for tension members, the allowable stress is

$$10000\left(1 + \frac{\text{min. stress}}{\text{max. stress}}\right) \text{lbs. per square inch.}$$

In the above case the allowable stress would be

$$10000\left(1 + \frac{50000}{150000}\right) = 13333 \text{ lbs. per square inch.}$$

Prob. 47. A deck Howe truss, for a railroad bridge, Fig. 28, has 12 panels, each 12 feet long, and 12 feet deep; the

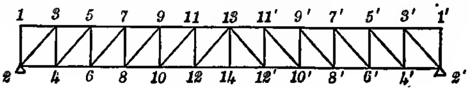


Fig. 28

dead load is given by formula (4), Art. 15, the live load is 1500 lbs. per linear foot per truss: find the maximum chord stresses.

Here we have the dead panel load per truss from formula (4)

$$= \frac{(6.5 \times 144 + 675)12}{2} = 9666 \text{ lbs.} = 4.833 \text{ tons, say 5 tons.}$$

$$\text{Live panel load per truss} = \frac{1500 \times 12}{2000} = 9 \text{ tons.}$$

We have then at each upper apex a load of $5+9=14$ tons, from which the maximum chord stresses may be computed directly.

Ans. Maximum stresses in lower chord

$= 77, 140, 189, 224, 245, 252$ tons.

The upper chord stresses can be written from the lower by (1) of Art. 5.

Prob. 48. A deck Howe truss has 11 panels, each 11 feet long, and 11 feet deep; the dead load is 400 lbs. per foot per truss, and the live load is 1200 lbs. per foot per truss: find the maximum chord stresses.

Ans. Lower chord stresses

$= 44.0, 79.2, 105.6, 123.2, 132, 132$ tons.

The upper chord stresses for the Howe and Pratt trusses equal the lower chord stresses with contrary signs, (1) of Art. 5; also, all the chord stresses in the Howe and Pratt are the same, no matter on which chord the loads may be placed.

Art. 22. Position of Uniform Live Load Causing Maximum Shears.—(1) For a uniform live load the maximum positive shear at any point N, Fig. 27, occurs when every possible load is added to the right and when there is no load on the left; for the shear at this point N is then equal to the left reaction R_1, and adding a load to the right increases the left reaction R_1 and therefore the positive shear, while adding a load to the left decreases the positive shear. Hence *the maximum positive shear at any point occurs when the live load extends from that point to the remote abutment.*

(2) Let a uniform live load be placed on the left of N, producing the left reaction R_1. Now this reaction R_1 is

only a part of the weight of the live load, while the shear is this reaction minus the whole weight of the load, and is therefore *negative;* and every load added to the left of this point increases numerically this negative shear. Hence *the maximum negative shear at any point occurs when the live load extends from that point to the nearest abutment.*

Prob. **49.** Find the greatest positive and negative shears for each panel of Fig. 27 when the apex live load is 9 tons.

By the principle just proved, the greatest positive live load shear in any panel occurs when all joints on the right are loaded, the joints on the left being unloaded. Hence, for greatest positive shear in 1–3 the truss is fully loaded. We have then

$$\text{Positive shear in } 1\text{–}3 = 9 \times 2\tfrac{1}{2} = 22.5 \text{ tons.}$$

There is no negative shear in 1–3.

For greatest shear in 3–5 all joints except 3 are loaded. Taking moments about right end, we have

$$\text{Positive shear in } \quad 3\text{–}5 = R_1 = 9(\tfrac{1}{6} + \tfrac{2}{6} + \tfrac{3}{6} + \tfrac{4}{6}) = 15 \text{ tons.}$$

$$\text{Negative shear in } \quad 3\text{–}5 = R_1 - 9 = 9 \times \tfrac{5}{6} - 9 = 7.5 - 9$$
$$= -1.5 \text{ tons,}$$

or, negative shear in $3\text{–}5 = -9 \times \frac{1}{6} = -1.5$ tons, as before.

Likewise,

$$\text{Positive shear in } \quad 5\text{–}7 = \tfrac{9}{6}(1 + 2 + 3) = 9 \text{ tons.}$$

$$\text{Negative shear in } \quad 5\text{–}7 = -\tfrac{9}{6}(1 + 2) = -4.5 \text{ tons.}$$

$$\text{Positive shear in } \quad 7\text{–}9 = \tfrac{9}{6}(1 + 2) = 4.5 \text{ tons.}$$

$$\text{Negative shear in } \quad 7\text{–}9 = -\tfrac{9}{6}(1 + 2 + 3) = -9.0 \text{ tons.}$$

$$\text{Positive shear in } \quad 9\text{–}11 = \tfrac{9}{6} \times 1 = 1.5 \text{ tons.}$$

$$\text{Negative shear in } \quad 9\text{–}11 = -\tfrac{9}{6}(1 + 2 + 3 + 4) = -15 \text{ tons.}$$

$$\text{Negative shear in } 11\text{–}13 = -\tfrac{9}{6}(1 + 2 + 3 + 4 + 5)$$
$$= -22.5 \text{ tons.}$$

There is no positive shear in 11–13.

The greatest positive shears are equal and symmetrical to the greatest negative shears. Thus, the positive shear in 1–3 is equal numerically to the negative shear in 11–13; the positive shear in 3–5 is equal numerically to the negative shear in 9–11; and so on.

Prob. **50.** Find the greatest positive and negative shears for each panel of Fig. 25 when the apex live load is 6 tons.

Ans. Positive shears = 21.0, 15.75, 11.25, 7.5, 4.5, 2.25, 0.75, 0.0 tons.

Negative shears = 0.0, 0.75, 2.25, 4.5, 7.5, 11.25, 15.75, 21.0 tons.

Prob. **51.** Find the maximum positive and negative shears for each panel of Fig. 28 when the apex dead and live loads are 5 and 9 tons, respectively, as in Prob. 47.

Ans.

Members.	2–4	4–6	6–8	8–10	10–12	12–14
Dead load shear	+27.50	+22.50	+17.50	+12.50	+ 7.50	+ 2.50
Live load positive shear	+49.50	+41.25	+33.75	+27.00	+21.00	+15.75
Live load negative shear	0.00	– 0.75	– 2.25	– 4.50	– 7.50	–11.25
Maximum shear	+77.00	+63.75	+51.25	+39.50	+28.50	+18.25
Minimum shear	+27.50	+21.75	+15.25	+ 8.00	0.00	– 8.75

The maximum shear is always positive in the left half of the truss, but the minimum may be either positive or negative. In this problem we see that a negative shear can occur in panel 12–14, and that there is no shear in panel 10–12, while the shears in all the other panels are positive. Members to the left of 10, and of course in the four right hand panels also, are therefore subjected to but one kind of stress, while in the four middle panels they may be sub-

jected to either kind of stress, and hence must be counterbraced. Though there is no negative shear in panel 10–12, it would be counterbraced for safety.

Art. 23. The Warren Truss.—The chord stresses in the Warren Truss may be computed by the method of chord increments, (2) of Art. 19, and the web stresses by the method of Arts. 18 and 22. The dead and live load stresses may be found separately and their sum taken for the maximum and minimum stresses; or the maximum and minimum stresses of any member may be determined directly, from a single equation, by placing the dead and live loads in proper position. In the following solution of the deck Warren truss the dead and live load stresses are found separately.

Prob. 52. A deck Warren truss, as a deck railroad bridge, Fig. 29, has 10 panels, each 12 feet long, its web members

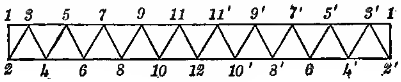


Fig. 29

all forming equilateral triangles (Art. 14); the dead load is given by formula (2), Art. 15, the live load is 1500 lbs. per foot per truss: find the maximum and minimum stresses in all the members.

The dead panel load per truss from formula (2)

$$= \frac{(7 \times 120 + 600)12}{2} = 8640 \text{ lbs.} = 4.32 \text{ tons}$$

$$= \text{say, 4 tons, for convenience in computation.}$$

$$\text{Live panel load per truss} = \frac{1500 \times 12}{2000} = 9 \text{ tons.}$$

Since the loads are distributed uniformly along the joints of the upper chord, 3 and 3′ will receive three fourths of a panel load each. The dead and live load stresses will be found separately. $\tan\theta = .577$; $\sec\theta = 1.1547$.

Dead Load Stresses.

Left reaction $= 4.75 \times 4 = 19$ tons. This is also the shear in panel 1–3. The dead load shear in each of the other panels is found by subtracting the loads on the left of the panel from the abutment reaction. We have then the following chord stresses by chord increments:

Upper Chord Stresses.

Stress in 3–5 $= (19 + 16) \times .577 = -20.20$ tons.

Stress in 5–7 $= (19 + 2 \times 16 + 12) \times .577 = -36.36$ tons.

Stress in 7–9 $= (19 + 2 \times 16 + 2 \times 12 + 8) \times .577$
$= -47.90$ tons.

Stress in 9–11 $= (75 + 2 \times 8 + 4) \times .577 = -54.82$ tons.

Stress in 11–11′ $= (91 + 2 \times 4) \times .577 = -57.12$ tons.

Stress in 1–3 $=$ 00.00 tons.

Lower Chord Stresses.

Stress in 2–4 $= 19 \times .577 = 10.96$ tons.

Stress in 4–6 $= (19 + 2 \times 16) \times .577 = 29.42$ tons.

Stress in 6–8 $= (51 + 2 \times 12) \times .577 = 43.28$ tons.

Stress in 8–10 $= (75 + 2 \times 8) \times .577 = 52.51$ tons.

Stress in 10–12 $= (91 + 2 \times 4) \times .577 = 57.12$ tons.

Web Stresses (by Art. 18).

Stress in 2–3 $= 19 \times 1.154 = -21.85$ tons.

Stress in 3–4 $= 16 \times 1.154 = +18.46$ tons $= -$stress in 4–5.

Stress in 5–6 $= 12 \times 1.154 = +13.84$ tons $= -$stress in 6–7.

Stress in 7–8 $= 8 \times 1.154 = +9.23$ tons $= -$stress in 8–9.

Stress in 9–10 $= 4 \times 1.154 = +4.61$ tons

$= -$stress in 10–11.

Stress in 11–12 $= 0.00$ $= -$stress in 12–12′.

Stress in 1–2 $= \frac{1}{4}$ of a panel load $= 1$ ton.

Live Load Stresses.

The maximum live load chord stresses occur when the live load covers the whole length of the truss (Art. 20); hence they are found in precisely the same way as the dead load chord stresses above; or we may find them by multiplying the above dead load chord stresses by the ratio of live to dead panel load, which $= 2\frac{1}{4}$ here, giving us the following chord stresses:

Upper Chord Stresses.	Lower Chord Stresses.
1–3 = 00.00 tons.	2–4 = 24.66 tons.
3–5 = − 45.45 tons.	4–6 = 66.19 tons.
5–7 = − 81.81 tons.	6–8 = 97.38 tons.
7–9 = − 107.77 tons.	8–10 = 118.15 tons.
9–11 = − 123.34 tons.	10–12 = 128.52 tons.
11–11′ = − 128.52 tons.	

Web Stresses.

The stress in any web member is equal to the shear in the section which cuts that member and two horizontal chord members, multiplied by the secant of the angle which the web member makes with the vertical (Art. 18); the stress is therefore a maximum when the shear is a maximum. The maximum positive live load shear in any panel occurs when all joints on the right are loaded and the joints on the left are unloaded (Art. 22).

The maximum positive live load shear for 2–3 will occur when the truss is fully loaded. We have then

$$\text{Stress in } 2\text{–}3 = 9 \times 4.75 \times 1.154 = -49.30 \text{ tons.}$$

The maximum positive shear for 3–4 and 4–5 will occur when all joints except 3 are loaded (Art. 22). Taking moments about the right end, and multiplying by sec θ, we have

Stress in 3–4

$$= \tfrac{9}{20}(\tfrac{3}{4} + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17) \times 1.1547$$
$$= \tfrac{9}{20} \times \tfrac{323}{4} \times 1.1547 = 41.96 \text{ tons} = -\text{stress in } 4\text{–}5.$$

Stress in 5–6

$$= \tfrac{9}{20}(\tfrac{3}{4} + 3 + 5 + 7 + 9 + 11 + 13 + 15) \times 1.1547$$
$$= 33.13 \text{ tons} = -\text{stress in } 6\text{–}7.$$

$$\text{Stress in } 7\text{–}8 = \tfrac{9}{20}(\tfrac{3}{4} + 3 + 5 + 7 + 9 + 11 + 13) \times 1.1547$$
$$= 25.33 \text{ tons} = -\text{stress in } 8\text{–}9.$$

$$\text{Stress in } 9\text{–}10 = \tfrac{9}{20}(\tfrac{3}{4} + 3 + 5 + 7 + 9 + 11) \times 1.1547$$
$$= 18.58 \text{ tons} = -\text{stress in } 10\text{–}11.$$

$$\text{Stress in } 11\text{–}12 = \tfrac{9}{20}(\tfrac{3}{4} + 3 + 5 + 7 + 9) \times 1.1547$$
$$= 12.87 \text{ tons} = -\text{stress in } 12\text{–}11'.$$

Stress in $11'-10' = \frac{9}{20}(\frac{3}{4} + 3 + 5 + 7) \times 1.1547$
$= 8.19$ tons $= -$ stress in $10'-9'$.

Stress in $9'-8' = \frac{9}{20}(\frac{3}{4} + 3 + 5) \times 1.1547$
$= 4.55$ tons $= -$ stress in $8'-7'$.

Stress in $7'-6' = \frac{9}{20}(\frac{3}{4} + 3) \times 1.1547$
$= 1.96$ tons $= -$ stress in $6'-5'$.

Stress in $5'-4' = \frac{9}{20}(\frac{3}{4}) \times 1.1547$
$= 0.40$ tons $= -$ stress in $4'-3'$.

Stress in $3'-2' = 0.00$ tons.

Collecting the above results we may enter them in a table as follows:

TABLE OF STRESSES IN ONE TRUSS.

Upper Chord Stresses.

Members.	1–3	3–5	5–7	7–9	9–11	11–11'
Dead load	00.00	−20.20	− 36.36	− 47.90	− 54.82	− 57.12
Live load	00.00	−45.45	− 81.81	−107.77	−123.34	−128.52
Max. stress	00.00	−65.65	−118.17	−155.67	−178.16	−185.64
Min. stress	00.00	−20.20	− 36.36	− 47.90	− 54.82	− 57.12

Lower Chord Stresses.

Members.	2–4	4–6	6–8	8–10	10–12
Dead load . .	+10.96	+29.42	+ 43.28	+ 52.51	+ 57.12
Live load . . .	+24.66	+66.19	+ 97.38	+118.15	+128.52
Max. stress . .	+35.62	+95.61	+140.66	+170.66	+185.64
Min. stress . .	+10.96	+29.42	+ 43.28	+ 52.51	+ 57.12

WEB STRESSES.

MEMBERS.	2–3	3–4	4–5	5–6	6–7
Dead load	−21.85	+18.46	−18.46	+13.84	−13.84
Live load. From right . . .	−49.30	+41.96	−41.96	+33.13	−33.13
Live load. From left	00.00	− 0.40	+ 0.40	− 1.96	+ 1.96
Max. stress	−71.15	+60.42	−60.42	+46.97	−46.97
Min. stress	−21.85	+18.06	−18.06	+11.88	−11.88
MEMBERS.	**7–8**	**8–9**	**9–10**	**10–11**	**11–12**
Dead load	+ 9.23	− 9.23	+ 4.61	− 4.61	+ 0.00
Live load. From right . . .	+25.33	−25.33	+18.58	−18.58	+12.87
Live load. From left	− 4.55	+ 4.55	− 8.19	+ 8.19	−12.87
Max. stress	+34.56	−34.56	+23.19	−23.19	+12.87
Min. stress	+ 4.68	− 4.68	− 3.58	+ 3.58	−12.87

We see from this table of *web stresses* that, (1) when the live load comes on from the right, all the web members in the left half of the truss are subjected to but one kind of stress, that is, the members 2–3, 4–5, 6–7, 8–9, 10–11 are subjected to compressive stresses, and the members 3–4, 5–6, 7–8, 9–10, 11–12 are subjected to tensile stresses; (2) when the live load comes on from the left, all the web members to the left of 9–10 are subjected to but one kind of stress, but the members 9–10, 10–11, 11–12 have their stresses changed, that is, the stresses in 9–10 and 11–12 are changed from tensile to compressive, while that in 10–11 is changed from compressive to tensile.

Hence, the members 2–3, 4–5, 6–7, 8–9, should be struts to carry compression only, and the members 3–4, 5–6, 7–8,

should be ties to carry tension only, while 9–10, 10–11, 11–12 should be members capable of resisting both compression and tension, and should therefore be *counter braces* (Art. 1): and the same is true for the right half of the truss.

In this solution for web stresses the live load was brought on from the *right*, and the maximum *positive* shear was found in each panel of the *right* half of the truss, and then the resulting stress; if preferred, the live load may be brought on from the *left*, and the maximum *negative* shear be found in each panel of the *left* half of the truss (Art. 22), and then the resulting stress. Thus, the maximum positive shear in any panel in the right half of the truss as $6'$–$8'$ has the same numerical value as the maximum negative shear in the corresponding member 6–8 in the left half; and the resulting stress in any member $7'$–$8'$ has the same value as that in the corresponding member 7–8.

The maximum and minimum stresses may be determined directly from a single equation by placing the dead and live loads in proper position.

Thus, for the maximum stress in 8–9, we pass a section cutting it, place the live load on the right, and have

$$\text{Max. stress in 8–9} = -[2\times 4 + \tfrac{9}{20}(\tfrac{3}{4}+3+5+7+9+11+13)] \times 1.1547 = -34.57 \text{ tons, as before.}$$

For the minimum stress the live load is reversed and covers the truss on the left of the section. Thus

$$\text{Min. stress in 8–9} = -[2\times 4 - \tfrac{9}{20}(\tfrac{3}{4}+3+5)] \times 1.1547 = -4.69 \text{ tons, as before.}$$

Prob. 53. A deck Warren truss, like Fig. 29, has 10 panels, each 10 feet long, its web members all forming equilateral triangles; the dead load is 800 lbs. per foot per truss, and the live load is 1600 lbs. per foot per truss; the

joints 3 and 3′ receive three fourths of a panel load each: find the maximum and minimum stresses in all the members.

Ans. UPPER CHORD STRESSES.

MEMBERS.	1–3	3–5	5–7	7–9	9–11	11–11′
Max. stresses	00.00	−60.60	−109.08	−143.76	−164.52	−171.48
Min. stresses	00.00	−20.20	− 36.36	− 47.92	− 54.84	− 57.16

LOWER CHORD STRESSES.

MEMBERS.	2–4	4–6	6–8	8–10	10–12
Max. stresses . . .	+32.88	+88.20	+129.84	+157.56	+171.48
Min. stresses . . .	+10.96	+29.40	+ 43.28	+ 52.52	+ 57.16

WEB STRESSES.

MEMBERS.	2–3	3–4	4–5	5–6	6–7
Max. stresses . . .	−65.52	+55.80	−55.80	+43.29	−43.29
Min. stresses . . .	−21.84	+18.12	−18.12	+12.09	−12.09
MEMBERS.	7–8	8–9	9–10	10–11	11–12
Max. stresses . . .	+31.78	−31.78	+21.13	−21.13	+11.46
Min. stresses . . .	+ 5.18	− 5.18	− 2.70	+ 2.70	−11.46

Prob. 54. A deck Warren truss, like Fig. 29, has 8 panels, each 15 feet long, its web members all forming equilateral triangles; the dead load is given by formula (2) of Art. 15, the live load is 1600 lbs. per foot per truss, the joints 3 and 3′ receive three fourths of a panel load each: find the maximum and minimum stresses in all the members.

Ans. Max. stresses in upper chord
= 0.00, −67.68, −117.97, −148.25, −158.17 tons.

Min. stresses in upper chord
= 0.00, −21.00, −36.61, −46.01, −49.09 tons.

Max. stresses in lower chord
= +37.58, +97.96, +138.15, +158.17 tons.

Min. stresses in lower chord
= +11.66, +30.40, +42.87, +49.09 tons.

Max. web stresses
= −74.99, +60.68, −60.68, +43.43, −43.43, +27.57, −27.57, +13.68 tons.

Min. web stresses
= −23.27, +18.20, −18.20, +9.23, −9.23, −1.35, +1.35, −13.68 tons.

Prob. 55. A through Warren truss, Fig. 26, has 10 panels, each 12 feet long, its web members all forming equilateral triangles; the dead load is 500 lbs. per foot per truss, and the live load is 834 lbs. per foot per truss: find the maximum and minimum stresses in all the members.

Ans. Upper chord max.
= −41.52, −73.92, −96.96, −110.80, −115.44 tons.

Upper chord min.
= −15.57, −27.72, −36.36, −41.55, −43.29 tons.

Lower chord max.
= +20.80, +57.76, +85.44, +103.92, +113.12 tons.

Lower chord min.
= + 7.80, + 21.66, + 32.04, + 38.97, + 42.42 tons.

Web stresses max.
= −41.55, +41.55, −32.91, +32.91, −24.81, +24.81, −17.31, +17.31, −10.37, +10.37 tons.

Web stresses min.

$= -15.57, +15.57, -11.55, +11.55, -6.91, +6.91, -1.73, +1.73, +4.06, -4.06$ tons.

Prob. 56. A through Warren truss, like Fig. 26, has 8 panels, each 15 feet long, its web members all forming equilateral triangles; the dead load is 800 lbs. per foot per truss, and the live load 1600 lbs. per foot per truss: find the maximum and minimum stresses in all the members.

Ans. Upper chord max.

$= -48.48, -83.16, -103.92, -110.88$ tons.

Upper chord min.

$= -16.16, -27.72, -34.64, -36.96$ tons.

Lower chord max.

$= +24.24, +65.76, +93.60, +107.40$ tons.

Lower chord min.

$= +8.08, +21.92, +31.20, +35.80$ tons.

Web stresses max.

$= -48.48, +48.48, -35.68, +35.68, -24.20, +24.20, -13.80, +13.80$ tons.

Web stresses min.

$= -16.16, +48.48, -10.40, +10.40, -3.48, +3.48, +4.68, -4.68$ tons.

Prob. 57. A deck Warren truss, like Fig. 29, has 7 panels, each 15 feet long and 15 feet deep, its web members all forming isosceles triangles; the dead load is 4 tons, and the live load is 9 tons, per panel per truss, the first and last joints 3 and 3′ receiving three quarters of a panel load each, as in Probs. 51, 52, and 53: find the maximum and minimum stresses in all the members.

Ans. Upper chord max.

$= -0.00, -37.38, -63.38, -76.38$ tons.

Upper chord min.

$= - 0.00, \ - 11.50, \ - 19.50, \ - 23.50$ tons.

Lower chord max.

$= + 21.13, \ + 53.65, \ + 73.13, \ + 79.63$ tons.

Lower chord min.

$= + 6.50, \ + 16.50, \ + 22.50, \ + 24.50$ tons.

Web stresses max.

$= - 47.23, \ + 36.87, \ - 36.87, \ + 24.49, \ - 24.49, \ + 13.56, \ - 13.56$ tons.

Web stresses min.

$= - 14.53, \ + 10.64, \ - 10.64, \ + 4.01, \ - 4.01, \ - 4.05, \ + 4.05$ tons.

Art. 24. Mains and Counters. — In the Pratt truss, Fig. 30, all the verticals, except 1–4 and 1′–4′, are to take compression only, and all the inclined members, except 1–2,

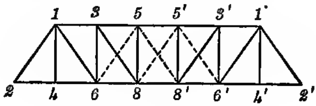


Fig. 30

1′–2′ are to take tension only. The two inclined members, 1–2 and 1′–2′, are usually called the "*inclined end posts.*" The verticals 1–4 and 1′–4′ do not form any part of the truss proper, since they serve only to carry the loads at 4 and 4′ up to the hip joints 1 and 1′; they are called *hip verticals.*

The members 1–6, 3–8, 3′–8′, and 1′–6′ are *mains,* or *main ties.* They are the only inclined ties that are put under stress by the action of the dead load, or by a uniform dead and live load extending over the whole truss. The members 5–6, 5–8′, 5′–8, and 5′–6′, in dotted lines, are *counters,*

or *counter ties.* In the Pratt truss there are no counter braces (Art. **1**). These four counters are called into play only when the live load covers a part of the truss.

Thus, if a load be placed at the joint 6′, it is carried by the truss to the abutments at 2 and 2′. The part of this load which goes to 2 may be conceived as being carried up to 5′, down to 8′, up to 5, down to 8, up to 3, down to 6, up to 1, down to the abutment at 2. The other part of this load, which goes to the right, passes up to 1′, then down to 2′. For this loading, the counters 5′–6′, 5–8′, and the mains 3–8, 1–6, and 1′–6′ are put under stress, while the counters 5′–8, 5–6, and the main tie 3′–8′ are idle, and might be removed without endangering the truss. If the two middle joints, 8 and 8′, are loaded equally, the part of the load at 8′ going to 2′ is just balanced by the part of the load at 8 going to 2, and hence there is no stress in the intermediate web members 5–8′, 5′–8, 5–8, and 5′–8′, nor in the counters 5–6 and 5′–6′. If the live load going from 6′ to the left abutment is greater than the dead load going from 8′ to the right abutment, the counter 5′–6′ must be inserted. If this counter 5′–6′ were not inserted, the panel 6′–8′ would be distorted. The load at 6′ would bring the opposite corners 3′ and 8′ nearer together, and the opposite corners 5′ and 6′ farther apart, because the main tie 3′–8′ cannot take compression. Whenever the live load would tend to cause compression in any main tie, a counter must be inserted uniting the other corners of the panel. Both diagonals in any panel cannot have compression or tension at the same time.

If the action of the dead and live loads tend to subject a member to stresses of opposite kinds, the resultant stress in the member will be equal to the numerical difference of the opposite stresses, and will be of the same kind as the greater. Thus, if the dead load, going from 8 to the left

abutment, should subject the main tie 3–8 to a tension of 8 tons, and if the live load, going from 6 to the right abutment, should tend to subject the same tie to a compression of 5 tons, there would be a resulting tension of 3 tons in the member 3–8; and the counter 5–6 would not be needed for this loading. But, if the dead load should subject the tie 3–8 to a tension of only 2 tons, while the the live load should tend to subject the same piece to a compression of 5 tons, there would be a resulting force of 3 tons, tending to bring the joints 3 and 8 nearer together, or to separate the joints 5 and 6 from each other. In order to provide for this compression or tension, either a brace would be needed alongside the main tie 3–8, to take the compression of 3 tons, or the counter tie 5–6 would be needed to take the tension of 3 tons.

In the Howe truss, Fig. 31, the verticals are to take tension only, and the inclined members are to take compression

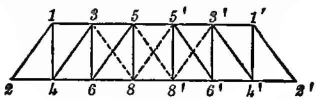


Fig. 31

only. The mains, or main braces, are represented by the full lines; the dotted lines denote counters, or counter braces. Under the action of the dead load, or of a uniform dead and live load covering the whole truss, the main braces are the only diagonals that are put under stress. The counter braces are called into play only when the live load covers a part of the truss, as in the case of the Pratt truss.

Thus, if a load be placed at the joint 6, the part of it which goes to the right abutment 2′ may be conceived as

being carried up to 3, down to 8, up to 5, down to 8′, up to 5′, down to 6′, up to 3′, down to 4′, up to 1′, down to 2′. If the live load going from 6 to the right abutment is greater than the dead load going from 8 to the left abutment, the counter brace 3–8 must be inserted. If this counter 3–8 were not inserted, the panel 6–8 would be distorted. The load at 6 would bring the opposite corners 3 and 8 nearer together, and the opposite corners 5 and 6 farther apart, because the main brace 5–6 cannot take tension. But if the live load going from 6 to the right abutment is *less* than the dead load going from 8 to the left abutment, the counter brace 3–8 will not be needed for this loading. The main and counter in any panel cannot both take stress at the same time by any system of loading.

We may determine where the counters are to begin, as follows: Consider the left half of the truss, and let the live load come on from the left; then the dead and live load shears, or stresses, are of opposite kinds. It results from this at once that counters — counter ties in the Pratt, and counter braces in the Howe — must begin in that panel in which the stress or shear caused by the live load is greater than that of the opposite kind caused by the dead load.

Thus, in Fig. 30, let the dead panel load be 3 tons and the live panel load 14 tons. Then we have the following maximum negative shears:

$$\text{Shear in } 2\text{–}4 = +9 - 0 \qquad\qquad = +9.$$

$$\text{Shear in } 4\text{–}6 = +6 - \tfrac{1}{7} \times 14 \qquad = +4.$$

$$\text{Shear in } 6\text{–}8 = +3 - 14\left(\tfrac{1}{7} + \tfrac{2}{7}\right) = -3.$$

Hence the counters must begin in the third panel, and therefore the third, fourth, and fifth panels must have counter ties for this loading.

Prob. 58. In the Howe truss of Fig. 28, with 12 panels, the dead panel load being 4 tons and the live 9 tons, find the number of panels to be counterbraced.

Ans. The 5th, 6th, 7th, and 8th panels must have counter braces.

Art. 25. The Howe Truss. — The maximum and minimum stresses, both in the chords and the web members of the Howe truss, may now be computed by the principles of Arts. 18, 19, 22, and 24. The dead and live load stresses may be found separately, and their sum taken for the maximum and minimum stresses; or, the maximum and minimum stresses of any member may be determined directly, from a single equation, by placing the dead and live loads in proper position. In the following solution of the through Howe truss, this method is the one employed:

Prob. 59. A through Howe truss, as a through railroad bridge, Fig. 32, has 10 panels, each 12 feet long and 12 feet

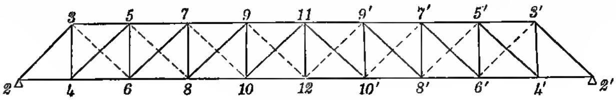


Fig. 32

deep; the dead load is given by formula (4), Art. 15, the live load is 1500 lbs. per foot per truss: find the maximum and minimum stresses in all the members.

From formula (4), Art. 15, we find the dead panel load per truss

$$= \frac{(6.5 \times 120 + 675)\,12}{2} = 8730 \text{ lbs.} = 4.365 \text{ tons}$$

$$= \text{say, 4 tons, for convenience of computation.}$$

$$\text{Live panel load per truss} = \frac{1500 \times 12}{2000} = 9 \text{ tons.}$$

$$\tan\theta = 1, \ \sec\theta = 1.41.$$

The maximum and minimum *chord stresses* may be written out at once (Arts. 19 and 21). They are as follows:

Max. stresses in lower chord = 58.5, 104.0, 136.5, 156.0, 162.5 tons.

Min. stresses in lower chord = 18.0, 32.0, 42.0, 48.0, 50.0 tons.

The upper chord stresses may be written from the lower at once. Thus, stress in 3–5 = – stress in 2–4; and so on.

Web Stresses.

The maximum stress in any web member is equal to the maximum shear in the section which cuts that member and two horizontal chord members, multiplied by the secant of the angle which the web member makes with the vertical (Art. 18). The maximum positive live load shear in any panel in the left half of the truss occurs when all joints on the right are loaded and the joints on the left are unloaded (Art. 22). For the maximum stress in 2–3, we pass a section cutting it, place the live load on the right, in this case covering the whole truss, and have

Max. stress in 2–3

$$= -[4 \times 4.5 + \tfrac{9}{10}(1+2+3+4+5+6+7+8+9)]1.41$$
$$= -82.48 \text{ tons.}$$

For the max. stress in 4–5 all the joints on the right of 4 are loaded. Hence,

Max. stress in 4–5

$$= -[4 \times 3.5 + \tfrac{9}{10}(1+2+3+4+\cdots+8)]1.41 = -65.42 \text{ tons.}$$

Max. stress in 6–7

$$= -[4 \times 2.5 + \tfrac{9}{10}(1+2+3+\cdots+7)]1.41 = -49.63 \text{ tons.}$$

Max. stress in 8–9

$$= -[4 \times 1.5 + \tfrac{9}{10}(1+2+3+\cdots+6)]1.41 = -35.10 \text{ tons.}$$

Max. stress in 10–11

$$= -[4 \times 0.5 + \tfrac{9}{10}(1+2+\cdots+5)]1.41 = -21.85 \text{ tons.}$$

For the minimum stress in any member in the left half of the truss, the live load is reversed and covers the truss on the left of the section (Art. 22). Hence,

Min. stress in 2–3 $= -[4 \times 4.5 - 0]\,1.41 = -25.38$ tons.

Min. stress in 4–5 $= -[4 \times 3.5 - \frac{9}{10} \times 1]\,1.41 = -18.47$ tons.

Min. stress in 6–7

$$= -[4 \times 2.5 - \tfrac{9}{10}(1 + 2)]1.41 = -10.29 \text{ tons.}$$

Min. stress in 8–9

$$= -[4 \times 1.5 - \tfrac{9}{10}(1 + 2 + 3)]1.41 = -0.85 \text{ tons.}$$

Min. stress in 10–11

$$= -[4 \times 0.5 - \tfrac{9}{10}(1 + 2 + 3 + 4)]1.41 = +9.87 \text{ tons.}$$

That is, the minimum stress in 10–11 is a *tension* of 9.87 tons. But as the diagonals in the Howe truss are to be subjected only to *compression*, the brace 10–11 cannot take tension, the counter brace 9–12 must therefore be inserted to take this 9.87 tons, which is the excess of that part of the live load going from ~~8~~ to the right abutment over that part of the dead load going from ~~10~~ 12 to the left abutment. If a *tie* were placed alongside the main brace 10–11, it would take this tension of 9.87 tons, and the counter brace 9–12 would not be needed; or, if the diagonal 10–11 were built so as to take either tension or compression, the counter 9–12 would not be needed. In this latter case the member 10–11 would have a range of stress from -21.85 tons to $+9.87$ tons, the former when the live load covered all the joints on the right of it, the latter when it covered all the joints on the left. But as the diagonals are to be subjected only to compression in this truss, the minimum stress in 10–11 is zero, and the *tension* of 9.87 tons in 10–11 becomes the maximum *compression* in the counter brace 9–12. The three diagonals, 4–5, 6–7, 8–9, are always in compression.

Hence, theoretically, there is only one counter brace needed in the left half of this truss; but, practically, a counter brace would be put in the fourth panel also, so that the truss would have counter braces in its four middle panels.

The maximum stress in any vertical, except the middle one, is equal to the maximum positive shear in the section cutting it and two chord members, or it is equal to the maximum positive shear in the panel *toward* the abutment from the vertical. Thus,

Max. stress in 3–4 $= 13 \times 4.5 = 58.5$ tons.

Max. stress in 5–6

$$= [4 \times 3.5 + .9(1 + 2 + \cdots + 8)] = 46.4 \text{ tons.}$$

Max. stress in 7–8

$$= [4 \times 2.5 + .9(1 + 2 + \cdots + 7)] = 35.2 \text{ tons.}$$

Max. stress in 9–10

$$= [4 \times 1.5 + .9(1 + 2 + \cdots + 6)] = 24.9 \text{ tons.}$$

The maximum stress in the middle vertical 11–12 is eqnal, either to the maximum positive shear in panel 10–12, or to a full panel dead and live load, whichever is the greater. For, if this maximum positive shear is greater than a panel load, then the shear in the next right hand panel under the same loading is also positive, and the counter brace in that panel *will be in action*, thus making the shear in panel 10–12 the same as that in the vertical 11–12; but, when the counters are *not in action*, the stress in 11–12 is always equal to the load at 12.

Max. positive shear in 10–12

$$= 4 \times 0.5 + \tfrac{9}{10}(1 + 2 + \cdots + 5) = 15.5 \text{ tons.}$$

Full panel load $= 4 + 9 = 13$ tons.

$\therefore$ Max. stress in 11–12 $= 15.5$ tons.

The minimum stress in any vertical is equal to the maximum negative shear in the section cutting it and two chord members, or it is equal to the maximum negative shear in the panel toward the abutment from the vertical, *or to a dead panel load,* whichever is the greater. Thus,

Min. stress in 3–4 $= 4 \times 4.5 - 0 = 18.0$ tons.

Min. stress in 5–6 $= 4 \times 3.5 - \frac{9}{10} \times 1 = 13.1$ tons.

Min. stress in 7–8 $= 4 \times 2.5 - \frac{9}{10}(1 + 2) = 7.3$ tons.

Min. stress in 9–10 $= 4 \times 1.5 - \frac{9}{10}(1 + 2 + 3) = 0.6$ tons.

But the stress in a vertical of this truss cannot be less than the weight of a dead panel load; therefore

Min. stress in 9–10 = 4 tons.

Min. stress in 11–12 = 4 tons.

We may now collect these web stresses in a table as follows:

WEB STRESSES.

MEMBERS.	2–3	4–5	6–7	8–9	10–11	9–12
Max. stresses	−82.48	−65.42	−49.63	−35.10	−21.85	−9.87
Min. stresses	−25.38	−18.47	−10.29	− 0.85	0.00	0.00

VERTICALS.

MEMBERS.	3–4	5–6	7–8	9–10	11–12
Max. stresses	58.5	46.4	35.2	24.9	15.5
Min. stresses	18.0	13.1	7.3	4.0	4.0

In building Howe trusses of timber, it is usually the practice to put in all the dotted diagonals to keep the truss rigid. This is not done in metallic trusses.

Prob. **60.** A through Howe truss has 11 panels, each 10 feet long and 10 feet deep; the dead load is 800 lbs., and the live load is 1600 lbs. per foot per truss: find the maximum and minimum stresses in all the members.

Ans. Max. stresses in lower chord
$= 60, 108, 144, 168, 180$, and 180 tons.

Min. stresses in lower chord
$= 20, 36, 48, 56, 60$, and 60 tons.

Max. stresses in main braces
$= -84.60, \ -68.71, \ -53.83, \ -40.0, \ -27.18,$
-15.37 tons.

Min. stresses in main braces
$= -28.28, -21.53, -13.85, -5.13, -0.00, -0.00$ tons.

Max. stresses in counters $= -4.31, \ -15.38$ tons.

Max. stresses in verticals
$= +60.0, +48.73, +38.18, +28.36, +19.28$ tons.

Min. stresses in verticals
$= +20.0, +15.27, +9.82, +4.00, +4.00$ tons.

Prob. **61.** A deck Howe truss has 12 panels, each 10 feet long and 10 feet deep; the dead load is 600 lbs., and the live load is 1200 lbs. per foot per truss: find the maximum and minimum stresses in all the members.

Ans. Max. stresses in lower chord
$= 49.5, 90.0, 121.5, 144.0, 157.5, 162$ tons.

Max. stresses in main braces
$= -69.79, -57.81, -46.53, -35.95, -26.08, -16.9$ tons.

Min. stresses in main braces
$= -23.25, -18.33, -12.69, -6.35, 0.0, 0.0$ tons.

Max. stresses in counters $= -0.71, -8.46$ tons.

Max. stresses in verticals
$= 4.5, 41.0, 33.0, 25.5, 18.5, 12.0, 7.5$ tons.

The maximum stress in any vertical of the deck Howe truss except the middle one, is equal to the maximum positive shear in the section cutting it and two chord members, or it is equal to the maximum positive shear in the panel *away* from the abutment from the vertical. The stresses in the verticals need to be very carefully tested.

Prob. 62. A deck Howe truss has 10 panels, each 14 feet long and 14 feet deep; the dead load is 714 lbs. and the live load is 2000 lbs. per foot per truss: find the maximum and minimum stresses in all the members.

Ans. Max. stresses in lower chord
$= 85.5, 152.0, 199.5, 228.0, 237.5$ tons.

Max. in main braces
$= -120.55, -95.74, -72.89, -52.02, -33.37$ tons.

Min. in main braces
$= -31.73, -22.70, -11.70, 0.0, 0.0$ tons.

Max. in verticals
$= 9.5, 67.9, 51.7, 36.9, 23.5, 14.0$ tons.

Max. in counters $= -1.27, -16.22$ tons.

Prob. 63. A through Howe truss has 8 panels, each 18 feet long and 24 feet deep; the dead load is 806 lbs., and the live load is 1500 lbs. per foot per truss: find the maximum and minimum stresses in all the members.

Art. 26. The Pratt Truss. — All parts of the Pratt truss are of iron or steel, the verticals being compression members, except the hip verticals, and the diagonals tension members. The maximum and minimum stresses of any member may be determined directly from a single equation, as in the case of the Howe truss (Art. 25).

Prob. 64. A through Pratt truss, as a through railroad bridge, Fig. 33, has 10 panels, each 14 feet long and 14 feet

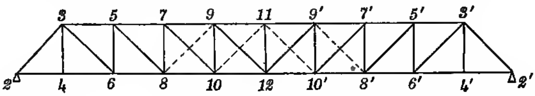


Fig. 33

deep; the dead load from formula (1), Art. 15, the live load is 1800 lbs. per foot per truss: find the maximum and minimum stresses in all the members.

From formula (1), Art. 15, the dead panel load per truss

$$= \frac{(9 \times 140 + 520)14}{2} = 12{,}460 \text{ lbs.} = 6.23 \text{ tons} = \text{say, } 6 \text{ tons.}$$

Live panel load per truss $= 16.2$ tons $=$ say, 16 tons.

$$\tan\theta = 1, \ \sec\theta = 1.414.$$

The max. and min. chord stresses may be written out at once (Arts. 19 and 21). They are the following:

Max. stresses in lower chord

$= 99.0, 99.0, 176.0, 231.0, 264.0$ tons.

Min. stresses in lower chord

$= 27.0, 27.0, 48.0, 63.0, 72.0$ tons.

WEB STRESSES.

The web stresses may be found exactly as in Prob. 60. The following are the equations for a few of the members:

Max. stress in 2–3 $= -[4.5 \times 22]1.41 = -139.5$ tons.

Max. stress in 3–6

$= [3.5 \times 6 + 1.6(1 + 2 + \cdots + 8)]1.41 = 110.8$ tons.

Max. stress in 5–8

$= [2.5 \times 6 + 1.6(1 + 2 + \cdots + 7)]1.41 = 84.3$ tons.

Max. stress in 7–8

$= -[1.5 \times 6 + 1.6(1 + 2 + \cdots + 6)] = -42.6$ tons.

Max. stress in 11–12

$= -[-0.5 \times 6 + 1.6(1 + 2 + \cdots + 4)] = -13.0$ tons.

Min. stress in 5–8

$= [2\frac{1}{2} \times 6 - 1.6(1 + 2)]1.41 = 14.4$ tons.

Max. stress in 10–11

$= [-\frac{1}{2} \times 6 + 1.6(1 + 2 + \cdots + 4)]1.41 = 18.5$ tons.

Thus we find the stresses in the following table:

WEB STRESSES.

MEMBERS.	2–3	3–6	5–8	7–10	9–12	8–9	10–11
Max. stresses	−139.5	+110.8	+84.3	+60.0	+38.0	+0.8	+18.5
Min. stresses	− 38.1	+ 27.4	+14.4	0.0	0.0	0.0	0.0

Max. stresses in the verticals $= +22.0$, -59.8, -42.6, -27.0, -18.5 tons.

Prob. 65. A through Pratt truss has 11 panels, each 10 feet long and 10 feet deep; the dead load is 1200 lbs., and the live load is 2000 lbs. per foot per truss: find the stresses in all the members.

Ans. Max. stresses in lower chord

$= 80, 80, 144, 192, 224, 240$ tons.

Max. in main diagonals

$= -113.1, +91.5, +71.5, +52.8, +35.4, +19.2$ tons.

Min. in main diagonals

$= -42.3, +32.6, +21.5, +9.2, 0.0, 0.0$ tons.

Max. in counters $= +4.4, +19.21$ tons.

Max. in verticals

$= +16.0, -50.7, -37.4, -25.1, -13.6$ tons.

Prob. 66. A deck Pratt truss, Fig. 34, has 10 panels, each 12 feet long and 12 feet deep; the dead load is 833

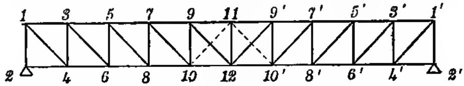


Fig. 34

lbs., and the live load is 1667 lbs. per foot per truss: find the stresses in all the members.

Ans. Max. stresses in lower chords

$= 0.0, 67.5, 120.0, 157.5, 180.0$ tons.

Max. in main ties $= 95.1, 75.4, 57.1, 40.2, 24.6$ tons.

Min. in main ties $= 31.7, 23.2, 13.4, 2.1, 0.0$ tons.

Max. in counters $= 10.6$ tons.

Max. in verticals

$= -75.0, -67.5, -53.5, -40.5, -28.5, -20.0$ tons.

Prob. 67. A deck Pratt truss has 9 panels, each 18 feet long and 24 feet deep; the dead load is 500 lbs., and the live load is 1000 lbs. per foot per truss: find the stresses in all the members.

Ans. Max. stresses in lower chords
$= 0.0, 40.5, 709.91, 91.1, 101.3$ tons.

Max. stresses in main ties
$= 67.5, 51.9, 37.5, 24.4, 12.5$ tons.

Min. stresses in main ties
$= 22.5, 15.6, 7.5, 0.0, 0.0$ tons.

Max. stresses in counters $= 1.9, 12.5$ tons.

Max. stresses in verticals
$= -60.8, -54.0, -41.5, -30.0, -19.5$ tons.

The chord stresses are the same in the Howe trusses, and also in the Pratt trusses, whether the load be placed on the top or the bottom chord.

Art. 27. The Warren Truss with Vertical Suspenders.—The left half of this truss is represented in Fig. 35, in which the web bracing consists of equilateral or

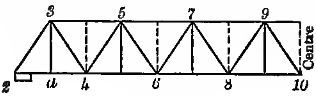


Fig. 35

isosceles triangles and vertical ties or suspenders. In a through truss, each of the verticals 3, 5, 7, 9, simply carries a panel load from the lower chord to the upper, while the verticals 4, 6, 8, 10, are only to support and stiffen the upper chord. If the truss is a deck, the last named verticals become posts.

Prob. 68. A through Warren truss with vertical ties, one half of which is shown in Fig. 35, has 16 panels, each 8 feet long, its braces all forming equilateral triangles; the dead load is given by formula (2) of Art. 15, the live load is 1500 lbs. per foot per truss: find the maximum and minimum stresses in all the members.

The dead panel load per truss

$$=\left(\frac{7\times 128+600}{2}\right)8=5984 \text{ lbs.}=3 \text{ tons.}$$

Live panel load per truss $=\dfrac{1500\times 8}{2000}=6$ tons.

$$\tan\theta=.577;\ \sec\theta=1.1547.$$

The chord stresses may be written out at once by chord increments.

For the verticals the maximum stress is evidently the dead and live panel load, or 9 tons; the minimum stress is the dead panel load, or 3 tons.

The following are the equations for determining the stresses in a few of the members:

Max. stress in 2–4 $=7.50\times 9\times .577=38.9$ tons.

Max. stress in 4–6

$=[7.50+6.50+5.50]9\times .577=101.3$ tons.

Max. stress in 3–5

$=-[7.50+6.50]\times 9\times .577=-72.7$ tons.

Max. stress in 2–3 $=-[7.50\times 9\times 1.154]=-77.9$ tons.

Max. stress in 5–6

$=[4.5\times 3+\frac{6}{16}(1+2+3+\cdots+12)]\,1.154$

$=42.75\times 1.154=+49.3$ tons.

Min. stress in 7–8

$=[2.5\times 3-\frac{6}{16}(1+2+3+\cdots+5)]\,1.154=+2.2$ tons.

In this way all the stresses may be found.

CHORD STRESSES.

MEMBERS.	3–5	5–7	7–9	9–9	2–4	4–6	6–8	8–10
Max. stresses	−72.7	−124.6	−155.8	−166.2	+38.9	+101.3	+142.8	+163.6
Min. stresses	−24.2	− 41.5	− 51.9	− 55.4	+13.0	+ 33.8	+ 47.6	+ 54.[illegible]

WEB STRESSES.

MEMBERS.	2–3	3–4	4–5	5–6	6–7	7–8	8–9	9–10
Max. stresses	−77.9	+67.9	−58.4	+49.3	−40.6	+32.5	−24.7	+17.3
Min. stresses	−26.0	+22.1	−17.7	+13.0	− 7.8	+ 2.2	+ 3.9	−10.4

We see from this table of web stresses that the members 8–9 and 9–10 should be capable of resisting both tension and compression, and hence should be counter braces (Art. 1); and, of course, the same is true for the right half of the truss.

Prob. 69. A through Warren truss with vertical ties, like Fig. 35, has 20 panels, each 10 feet long, its braces all forming equilateral triangles; the dead load is given by formula (2) of Art. 15, the live load is 1600 lbs. per foot per truss: find the maximum and minimum stresses in all the members.

Ans. Max. stresses in upper chord

$= -135.0, -240.0, -315.0, -360.0, -375.0$ tons.

Max. stresses in lower chord

$= +71.3, +191.3, +281.3, +341.3, +371.3$ tons.

Max. web stresses

$= -142.5, +128.0, -113.9, +100.2, -87.1, +74.4, -62.2, +50.4, -39.1, +28.3$ tons.

Min. web stresses

$= -54.8, +48.6, -41.9, +34.7, -27.1, +19.0, -10.5, +1.5, +8.0, -17.9$ tons.

Prob. 70. A through Warren truss with vertical ties, like Fig. 35, has 10 panels, each 12 feet long and 12 feet deep, its braces all forming isosceles triangles; the dead and live loads are 1000 lbs., and 2000 lbs. per foot per truss: find the maximum and minimum stresses in all the members.

Art. 28. The Double Warren Truss, or *Double Triangular Truss,* or *Girder.* — This truss, shown in Fig. 36, has two systems of triangular bracing, the one system represented in full lines and the other in dotted lines, the

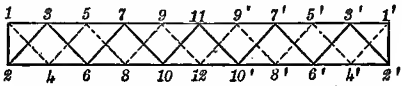


Fig. 36

chords being common to both systems. The roadway may be carried either upon the upper or the lower chord.

In all the trusses hitherto considered, a vertical section taken at any point of the truss will cut only three members; but in the double Warren truss this is not the case, since a vertical section will in general cut *four* members, causing the problem at first to seem to be *indeterminate* (Art. 7). This is obviated, however, by the fourth condition that: the truss is equivalent to the two trusses, shown in full and in dotted lines, welded into one. Thus, the loads at 6, 10, 10′, 6′ are carried to the abutments by the diagonals drawn in full, while the loads at 4, 8, 12, 8′, 4′ are carried by the dotted diagonals. The chord and web stresses are readily found as in a simple triangular truss, assuming each system as independent. This truss is used both as a riveted, and as a pin-connected bridge.

Prob. 71. A double Warren truss, Fig. 36, as a deck railroad bridge, has 10 panels, each 14 feet long and 14 feet deep; the dead load is given by formula (2) of Art. 15, the live load is 2000 lbs. per foot per truss: find the stresses in all the members.

The dead panel load per truss

$$= \left(\frac{7 \times 140 + 600}{2}\right) 14 = 11{,}060 \text{ lbs.} = 5.5 \text{ tons.}$$

Live panel load per truss $= 14$ tons.

$$\tan\theta = 1, \ \sec\theta = 1.414.$$

The following are the equations for determining the stresses in a few of the members:

Max. stress in 1–3 $= -2 \times 19.5 \times 1 = -39$ tons.

Max. stress in 3–5 $= -[2.0 + 2.5 + 1.5]19.5 = -117$ tons.

Max. stress in 6–8 $= [2.5 + 4.0 + 3.0]\,19.5 = +185.2$ tons.

Max. stress in 3–6

$= [1.5 \times 5.5 + 1.4\,(1 + 3 + 5 + 7)]\,1.414 = +43.3$ tons.

Min. stress in 5–8 $= [5.5 - 1.4 \times 2]\,1.414 = +3.8$ tons.

Thus the following stresses are determined:

UPPER CHORD STRESSES.

MEMBERS.	1–3	3–5	5–7	7–9	9–11
Max. stresses . . .	−39.0	−117.0	−175.5	−214.5	−234.0
Min. stresses . . .	−11.0	− 33.0	− 49.5	− 60.5	− 66.0

LOWER CHORD STRESSES.

MEMBERS.	2–4	4–6	6–8	8–10	10–12
Max. stresses . . .	+48.8	+126.8	+185.2	+224.3	+243.8
Min. stresses . . .	+13.8	+ 35.8	+ 52.3	+ 63.3	+ 68.8

WEB STRESSES (DOTTED SYSTEM).

MEMBERS.	1–4	4–5	5–8	8–9	9–12
Max. stresses	+55.2	−55.2	+31.5	−31.5	+11.9
Min. stresses	+15.6	−15.6	+ 3.8	− 3.8	−11.9

WEB STRESSES (FULL SYSTEM).

MEMBERS.	2–3	3–6	6–7	7–10	10–11
Max. stresses	−69.0	+43.3	−43.3	+21.7	−21.7
Min. stresses	−19.4	+ 9.7	− 9.7	− 4.1	+ 4.1

Prob. 72. A double deck Warren truss, like Fig. 36, has 12 panels, each 10 feet long and 10 feet deep; the dead load is 800 lbs., and the live load is 1600 lbs. per foot per truss: find the stresses in all the members.

Ans. Max. stresses in upper chord
$= -30, -90, -138, -174, -198, -210$ tons.

Max. stresses in lower chord
$= 36, 96, 144, 180, 204, 216$ tons.

Max. stresses in dotted diagonals
$= +42.4, -42.4, +27.3, -27.3, +14.1, -14.1$ tons.

Max. stresses in full diagonals
$= -50.9, +34.9, -34.9, +20.7, -20.7, +8.5$ tons.

Min. stresses in dotted diagonals
$= +14.1, -14.1, +6.6, -6.6, -2.9, +2.9$ tons.

Min. stresses in full diagonals
$= -17.0, +10.4, -10.4, +1.7, -1.7, -8.5$ tons.

Prob. 73. A double through Warren truss, like Fig. 36, has 10 panels, each 10 feet long and 10 feet deep; the dead and live loads are 800 lbs., and 2000 lbs. per foot per truss: find the stresses in all the members.

Ans. Max. stresses in upper chord
$= -35, -91, -133, -161, -175$ tons.

Max. stresses in lower chord
$= +28, +84, +126, +154, +168$ tons.

Max. stresses in diagonal ties
$= +49.4, +39.6, +31.1, +22.5, +15.6, +84$ tons.

Min. stresses in diagonal ties
$= +14.1, +11.3, +7.1, +2.8, -2.9, -8.4$ tons.

Prob. 74. A double through Warren truss, like Fig. 36, has 12 panels, each 15 feet long and 15 feet deep; the dead load is given by formula (3) of Art. 15, the live load is 2000 lbs. per foot per truss: find the stresses in all the members.

Art. 29. The Whipple Truss.—This truss, shown in Fig. 37, consists of two simple Pratt trusses combined.

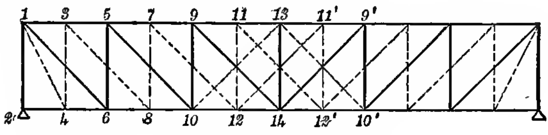


Fig. 37

The ties in the web system extend over two panels, and it is therefore often called a "double intersection Pratt truss."* The advantage over the Pratt for long spans is that it has short panels, while keeping the inclination of the diagonals at about 45°.

* Called also the Linville Truss.

This truss was at one time more common than any other in American bridge practice. It is still used though very rarely for highway bridges; for railway bridges it is as a rule avoided by the best practice.

It is assumed that this truss is equivalent to the two trusses shown in full and in dotted lines, and that each system acts independently and carries only its own loads to the abutments; but in the case of the web members, this is not *strictly* true. If, however, the truss is built with an even number of panels, the error in the web stresses obtained on the assumption of independent systems is probably very small. With the chord stresses there is no ambiguity; the chord stresses are a maximum for a full load, and may be found as usual by chord increments, or by moments. The vertical shear will be the same whether the load is applied at the top or at the bottom chord.

Prob. 75. A through Whipple truss, Fig. 37, has 12 panels, each 10 feet long and 20 feet deep; the dead load is 750 lbs., and the live load is 1200 lbs. per foot per truss: find the stresses in all the members.

The dead panel load per truss = 3.75 tons.

The live panel load per truss = 6.0 tons.

$$\tan\theta = 1: \ \sec\theta = 1.414: \ \tan\theta' = 0.5: \ \sec\theta' = 1.118.$$

The following are the equations for finding the stresses in a few of the members:

Max. stress in 4–6 $= 3 \times 9.75 \times 0.5 = 14.6$ tons.

Max. stress in 6–8 $= 14.6 + 2\frac{1}{2} \times 9.75 \times 1 = 39.0$ tons
$= -$ stress in 1–3.

Max. stress in 9–11
$= -[39.0 + (2 + 1\frac{1}{2} + 1 + \frac{1}{2})9.75] = -87.8$ tons
$=$ stress in 11–13,

since for a uniform load the diagonals meeting the upper chord between 9 and 9′ are not in action.

Max. stress in 1–4 $= 3 \times 9.75 \times 1.118 = 32.7$ tons.

Max. stress in 1–6

$$= [2\tfrac{1}{2} \times 3.75 + \tfrac{6}{12}(2+4+6+8+10)] \times 1.414$$
$$= 24.375 \times 1.414 = 34.5 \text{ tons.}$$

Max. stress in 1–2 $= -29.25 - 24.375 = -53.6$ tons.

Thus the following stresses are determined:

CHORD STRESSES.

MEMBERS.	2–4	4–6	6–8	8–10	10–12	12–14	9–13
Max. .	0.00	14.6	39.0	58.5	73.1	82.9	87.8
Min. .	0.00	5.6	15.0	22.5	28.1	31.8	33.8

STRESSES IN DIAGONALS.

MEMBERS.	1–4	1–6	3–8	5–10	7–12	9–14	11–12′	13–10′
Max. .	32.7	34.5	28.3	22.1	16.6	11.1	6.4	1.6
Min. .	12.6	13.3	9.9	6.5	2.5	0.0	0.0	0.0

STRESSES IN POSTS.

MEMBERS.	1–2	3–4	5–6	7–8	9–10	11–12	13–14
Max. .	53.6	20.0	15.6	11.8	7.9	4.5	1.1
Min. .	20.6	7.0	4.6	1.8	0.0	0.0	0.0

Prob. 76. A through Whipple truss, Fig. 38, has 16 panels, each 10 feet long and 20 feet deep; the dead and live loads are 800 lbs., and 1600 lbs. per foot per truss: find the stresses in all the members.

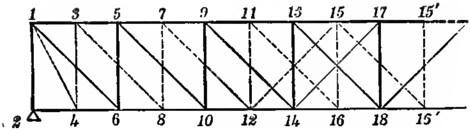


Fig. 38

Ans. CHORD STRESSES.

MEMBERS.	4–6	6–8	8–10	10–12	12–14	14–16	16–18	13–17
Max. .	24.	66.	102.	132.	156.	174.	186.	192.
Min. . .	8.	22.	34.	44.	52.	58.	62.	64.

STRESSES IN FULL DIAGONALS.

MEMBERS.	1–6	5–10	9–14	13–18	17–14'
Max. . . .	59.4	43.8	29.7	17.0	5.7
Min. . . .	19.8	12.7	4.2	0.0	0.0

STRESSES IN DOTTED DIAGONALS.

MEMBERS.	1–4	3–8	7–12	11–16	15–16'	15–12
Max. . .	53.7	51.6	36.8	23.3	11.3	0.7
Min. . .	22.6	16.3	8.4	0.0	0.0	0.0

STRESSES IN POSTS.

MEMBERS.	1–2	3–4	5–6	7–8	9–10	11–12	13–14	15–16	17–18
Max. . .	90.0	36.5	31.0	26.0	21.0	16.5	12.0	8.0	4.0
Min. . .	30.0	11.5	9.0	6.0	3.0	0.0	0.0	0.0	0.0

Prob. 77. A deck Whipple truss, like Fig. 38, has 16 panels, each 12 feet long and 24 feet deep; the dead load is 750 lbs. per foot per truss, and the live load is 2000 lbs. per foot per truss: find the stresses in all the members.

Art. 30. The Lattice Truss, or Quadruple Warren Truss.* — This truss, Fig. 39, contains four web systems

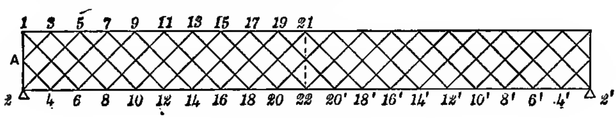


Fig. 39

welded into one. It is built only as a short-span riveted structure.

In determining the stresses in the different members of this truss, an ambiguity arises from two causes: (1) the web members being riveted together at each intersection, the different systems cannot act independently; and (2) two of the systems are not symmetrically placed in reference to the center of the truss. Thus, if equal loads be placed at each joint, the abutment at 2 will carry more than half of the load on the system 4, 12, 20, 16′, 8′, and less than half of the load on the system 8, 16, 20′, 12′, 4′. But it simplifies the solution to assume that each system acts independently, and is symmetrical in reference to the center of the truss. The chord and web stresses are then readily found, as in a simple triangular truss.

* All double systems, such as this and the Whipple, owing to the indeterminate character of the strains, are now usually avoided by the best practice.

Prob. 78. A through lattice truss containing four web systems, as shown in Fig. 39, has 20 panels, each 10 feet long, the depth of the truss is 20 feet; the dead load is given by formula (2) Art. 15, the live load is 2000 lbs. per foot per truss: find the stresses in all the members.

Dead panel load per truss = 5 tons.

Live panel load per truss = 10 tons.

$$\tan\theta = 1,\ \sec\theta = 1.414.$$

NOTE.—In finding the shear for a web member due to the dead load or to the live load covering the whole truss, a slight difficulty arises, owing to the two unsymmetrical systems above referred to. This shear may be found, either by taking the algebraic sum of the greatest positive and the greatest negative shears (Art. 22), or by simple inspection of the truss.

Thus, the dead load shear in 5–10 is the weight of the two panel loads at the joints 10 and 18, because the system to which 5–10 belongs is symmetrical with reference to the center of the truss; but the shear in 7–12 is not the weight of the two panel loads at the joints 12 and 20, because this system is not symmetrical in reference to the center. If the joint 20 were at the center 22, the shear for 7–12 would be $1\frac{1}{2}$ panel loads; but if it were at the joint 18, the shear would be 2 panel loads. Being midway between the joints 18 and 22, the shear for 7–12 is the mean of $1\frac{1}{2}$ and 2 panel loads, or 1.75 panel loads; and so for the shear in any other web member.

By chord increments, we have

Max. stress in 2–4 $= 2 \times 15 \times 1 = 30$ tons.

Max. stress in 4–6
$= (2 + 1.75 + 2.75) \times 15 = 97.5$ tons.

Max. stress in 6–8
$= 97.5 + (1.5 + 2.5) \times 15 = 157.5$ tons.

Max. stress in 1–3 $= 2.5 \times 15 = 37.5$ tons.

Max. stress in 3–5 $= (2.5 + 2 \times 2.25) \times 15 = 105.0$ tons.

Maximum Web Stresses.

The maximum stress in 3–8 will be when the dead and live loads cover the whole system to which 3–8 belongs. Likewise for the member 5–10. The maximum live load stress in 7–12 will be when the live panel loads are at the joints 12, 20, 16′, 8′, since these are the only loads which act in the system to which 7–12 belongs, on the right of 7–12. For the dead load stress, see Note.

Max. stress in 3–8

$= 2.25 \times 15 \times 1.414 = 47.7$ tons $= -$ stress in A–3.

Max. stress in 5–10

$= 2 \times 15 \times 1.414 = 42.4$ tons $= -$ stress in 2–5.

Max. stress in 7–12

$= [1.75 \times 5 + \frac{10}{20}(3+7+11+15)]1.414 = 37.8$ tons

$= -$ stress in 4–7.

Max. stress in 9–14

$= [1.5 \times 5 + \frac{1}{2}(2+6+10+14)]1.414 = 33.2$ tons

$= -$ stress in 6–9.

Thus the following stresses are determined:

Upper Chord Stresses.

Members.	1–3	3–5	5–7	7–9	9–11
Max. . . .	37.5	105.0	165.0	217.5	262.5
Min. . . .	12.5	35.0	55.0	72.5	87.5
Members.	**11–13**	**13–15**	**15–17**	**17–19**	**19–21**
Max. . . .	300.0	330.0	352.5	367.5	375.0
Min. . . .	100.0	110.0	117.5	122.5	125.0

LOWER CHORD STRESSES.

MEMBERS.	2–4	4–6	6–8	8–10	10–12
Max. . . .	30.0	97.5	157.5	210.0	255.0
Min. . . .	6.0	32.5	52.5	70.0	85.0
MEMBERS.	12–14	14–16	16–18	18–20	20–22
Max. . . .	292.5	322.5	345.0	360.0	367.5
Min. . . .	97.5	107.5	115.0	120.0	122.5

WEB STRESSES.

Max. stress in A–4 = 58.4, in 1–6 = 53.0, in 3–8 = 47.7, in 5–10 = 42.5, in 7–12 = 37.8, in 9–14 = 33.2, in 11–16 = 28.7, in 13–18 = 24.0, in 15–20 = 20.2, in 17–22 = 16.3, in 19–20′ = 12.3, in 21–18′ = 8.5, in 19′–16′ = 5.4 tons.

Min. stress in A–4 = 19.5, in 1–6 = 17.7, in 3–8 = 15.9, in 5–10 = 14.2, in 7–12 = 11.8, in 9–14 = 9.2, in 11–16 = 6.8, in 13–18 = 4.3, in 15–20 = 1.3, in 17–22 = – 2.1, in 19–20′ = – 5.3, in 21–18′ = – 8.5, in 19′–16′ = – 8.9 tons.

Max. stress in 1–A = 37.5, in A–2 = 112.5 tons.

Min. stress in 1–A = 12.5, in A–2 = 37.5 tons.

We see from these results that the 10 diagonals 14–17 and 17–22, 16–19 and 19–20′, 18–21 and 21–18′, 20–19′ and 19′–16′, 22–17′ and 17′–14′ require to be counterbraced.

Prob. 79. A deck lattice truss, containing four web systems, like Fig. 39, has 20 panels, each 12 feet long, the depth of truss is 24 feet; the dead and live loads are 500 lbs., and 1000 lbs. per foot per truss: find the stresses in all the members.

Ans. Max. stresses in upper chord = 18.0, 58.5, 94.5, 126.0, 153.0, 175.5, 193.5, 207.0, 216.0, 220.5 tons.

Max. stresses in lower chord = 22.5, 63.0, 99.0, 130.5, 157.5, 180.0, 198.0, 211.5, 220.5, 225.0 tons.

Max. stresses in struts *A*–3, 2–5, etc. = 35.0, 31.8, 28.6, 25.5, 22.7, 19.9, 17.2, 14.4, 12.1, 9.8, 7.4, 5.1, 3.2 tons.

Min. stresses in struts *A*–3, 2–5, etc. = – 11.7, – 10.6, – 9.5, – 8.5, – 7.0, – 5.5, – 4.1, – 2.6, – 0.8, + 1.2, + 3.2, + 5.1, + 7.4 tons.

Max. in *A*–1 = 22.5; min. in *A*–1 = 7.5. Max. in *A*–2 = 67.5; min. in *A*–2 = 22.5 tons.

Prob. 80. A through lattice truss, containing four web systems, like Fig. 39, has 16 panels, each 10 feet long, the depth of truss is 20 feet; the dead and live loads are 1000 lbs., and 2000 lbs. per foot per truss: find the stresses in all the members.

Art. 31. The Post Truss.* — This truss, shown in Fig. 40, is a special form of the double triangular truss (Art. 28). The members 3–4, 5–6, etc., are struts, and all

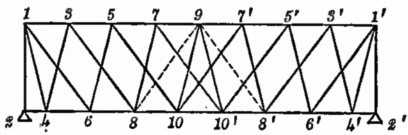


Fig. 40

the other diagonals are ties; the counters are shown in dotted lines. The distinctive feature of this truss is that the web struts, or posts, instead of standing vertically, are

* **This truss has become obsolete.**

inclined by one half of a panel length, and the ties by one and one half panel lengths. All the panels of both chords are of equal length, except the two end panels of the lower chord, which are each one half a panel in length.

There are ambiguities in the character of the stresses in this truss, as there are in all double systems of bracing. It is impossible to separate the two systems as they are connected at the center. But if we assume that, under a full load, the members 7–10′ and 7′–10 are not acting, if it is a deck truss, or that the members 7–10′, 7′–10, 9–10, and 9–10′ are not acting, if it is a through truss, then the chord stresses are readily found. Also, if we assume that the systems 2, 1, 6, 5, 10, 7′, 8′, 3′, 4′, 1′, 2′, and 2, 1, 4, 3, 8, 7, 10′, 5′, 6′, 1′, 2′ are independent, the web stresses may be readily found.

Prob. 81. A deck Post truss, Fig. 40, has 8 panels in the upper chord, each 10 feet long, and 20 feet deep; the dead and live loads are 400 lbs., and 1600 lbs. per foot per truss: find the stresses in all the members.

Dead panel load per truss = 2 tons.

Live panel load per truss = 8 tons.

$\tan\theta = 0.25$, and $\sec\theta = 1.0308$ for the posts.

$\tan\theta = 0.75$, and $\sec\theta = 1.25$ for the ties.

Max. stress in 1–3 $= [2 \times \frac{1}{4} + 1\frac{1}{2} \times \frac{3}{4}]10 = 16.3$ tons.

Max. stress in 4–6 $= [2 \times \frac{1}{4} + 2 \times \frac{1}{4}]10 = 10.0$ tons.

Max. stress in 5–6
$= [1\frac{1}{2} \times 2 + (2+4+6)] \times 1.03 = 15.5$ tons.

Max. stress in 3–8
$= [2 + (1 + 3 + 5)] \times 1.25 = 13.8$ tons.

Min. stress in 3–8 $= [2 - 8 \times \frac{1}{8}] \times 1.25 = 1.3$ tons.

Thus the following stresses are determined:

CHORD STRESSES.

MEMBERS.	1–3	3–5	5–7	7–9	4–6	C–3	8–10	10–10′
Max. . .	16.3	28.7	36.2	38.7	10.0	25.0	35.0	40.0
Min. . .	3.3	5.7	7.2	7.7	2.0	5.0	7.0	8.0

STRESSES IN TIES.

MEMBERS.	1–4	1–6	3–8	5–10	7–10′	8–9	C–7	4–5
Max. . .	20.6	18.7	13.8	8.7	7.5	5.0	2.5	1.3
Min. . .	4.2	3.8	1.3	0.0	0.0	0.0	0.0	0.0

STRESSES IN POSTS.

MEMBERS.	1–2	3–4	5–6	7–8	9–10	2–4
Max. . .	40.0	22.6	15.4	11.4	7.2	0.0
Min. . .	8.0	4.2	3.1	2.1	1.0	0.0

Prob. 82. A through Post truss, like Fig. 40, has 12 panels in the lower chord, each 10 feet long and 20 feet deep; the dead and live loads are 1000 lbs., and 2000 lbs. per foot per truss: find the stresses in all the members.

Art. 32. The Bollman Truss.—This truss, shown in Fig. 41, was the earliest type of iron truss built in the United States. It consists of a series of inverted king post trusses with unequally inclined ties, which carry the load at each joint directly to the ends of the upper chord. The vertical pieces are struts; the upper chord is in compression; and there is no stress in the lower chord. The short

ties shown in dotted lines in each panel serve to stiffen the structure.

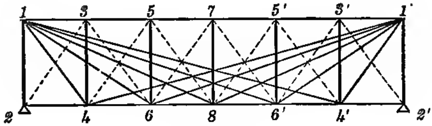


Fig. 41

Bollman trusses were largely used in the Baltimore and Ohio Railroad from 1840 to 1850; but they were abandoned long ago.

Prob. 83. A deck Bollman truss, Fig. 41, has 6 panels, each 12 feet long, and 16 feet deep; the dead and live loads are 1000 lbs., and 2000 lbs. per foot per truss: find the stresses in all the members.

Art. 33. The Fink Truss.—This truss, shown in Fig. 42, was first built about 1851, and was regarded as an improvement on the Bollman truss. It consists of a com-

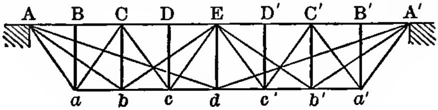


Fig. 42

bination of inverted king post trusses with equally inclined ties, as the primary system AdA', the secondary systems AbE and $A'b'E$, and the tertiary systems AaC, CcE, etc. The vertical pieces are struts, the upper chord is in compression, and there is no stress in the lower chord; all the diagonals are ties.

The load may be carried either upon the upper or the lower chord, though it has been used more often for deck

than for through bridges. These trusses were built from 1851 down to about 1876; but they are not now generally regarded with favor by bridge builders.

Prob. 84. A deck Fink truss, Fig. 42, has 8 panels, each 12 feet long, and 16 feet deep; the dead and live loads are 500 lbs., and 2000 lbs. per foot per truss: find the stresses in all the members.

The dead panel load per truss = 3 tons.

The live panel load per truss = 12 tons.

We see at once, from Fig. 42, that the maximum stresses in all the members occur when there is a full panel load at every apex. Hence the maximum stresses will be when 15 tons are placed at every apex, and the minimum stresses will be one fifth of the maximum. The maximum stress in each of the posts Ba, Dc is -15 tons. The maximum stress in the post Cb is the panel load of 15 tons at C, plus one half the panel load of 15 tons at D, plus one half the panel load of 15 tons at B, $= -30$ tons. Similarly the stress in $Ed = -60$ tons.

Also,

Stress in $Aa = \frac{1}{2} \times 15 \sec \theta = 9.4$ tons $= Ca = Cc = Ec$.

Stress in $Ab = \frac{1}{2} \times 30 \sec \theta_1 = 27.0$ tons $= Eb$.

Stress in $Ad = \frac{1}{2} \times 60 \sec \theta_2 = 95.0$ tons.

Stress in $AA' = -[\frac{1}{2} \times 15 \times \frac{3}{4} + \frac{1}{2} \times 30 \times \frac{6}{4} + \frac{1}{2} \times 60 \times \frac{12}{4}]$
$= -118.1$ tons.

Prob. 85. A deck Fink truss has 8 panels, each 10 feet long and 15 feet deep; the dead and live loads are 400 lbs. and 2000 lbs. per foot per truss: find the stresses in all the members.

BRIDGE TRUSSES WITH INCLINED CHORDS.

Art. 34. The Parabolic Bowstring Truss.—In this truss, Fig. 43, the lower chord is horizontal, and the upper

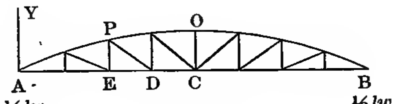


Fig. 43

chord joints lie on the arc of a parabola; the verticals may be in compression and the diagonals in tension, or the verticals may be in tension and the diagonals in compression.

Let $l =$ the length of the span AB, $d =$ the height of the arc OC at the center of the span, and $w =$ the uniform load in lbs. per foot of truss; let (x, y) be any joint P of the upper chord referred to the axes AB and AY, and S the stress in the lower chord panel opposite P. Then we have

$$S = \frac{(lx - x^2)w}{2y} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The equation of the parabola referred to its vertex O as origin is

$$(\tfrac{1}{2}l - x)^2 = 2p(d - y) \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and for the point A this becomes $\dfrac{l^2}{4} = 2pd$; $\therefore\ 2p = \dfrac{l^2}{4d}$. Substituting this in (2) and solving for y, we get

$$y = \frac{4d}{l^2}(lx - x^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

which in (1) gives

$$S = \frac{wl^2}{8d}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Hence, for a uniform load on a parabolic truss the stress in the lower chord is the same in every panel.

Since the horizontal component of the stress in any diagonal, as *PD* for example, is equal to the difference of the chord stresses in the adjacent panels, *ED* and *DC*.

Therefore, for a uniform load there is no stress in any diagonal.

It follows at once that, *for a uniform load the horizontal component of the stress in the upper chord is the same in every panel,* and is expressed by equation (4).

Therefore, for a uniform load the stress in any inclined panel of the upper chord is equal to the stress in a panel of the lower chord multiplied by the secant of the inclination.

Prob. 86. A parabolic bowstring, Fig. 44, as a through bridge, has 8 panels, each 10 feet long, and 10 feet center

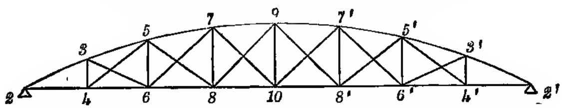


Fig. 44

depth, the verticals take either tension or compression, the diagonals are ties; the dead and live loads are 400 lbs., and 800 lbs. per foot per truss: find the stresses in all the members.

The max. stress in any panel of the lower chord, by (4),

$$= \frac{1200 \times \overline{80}^2}{80 \times 2000} = 48 \text{ tons.}$$

$$\therefore \text{ Min. stress} = \tfrac{1}{3} \times 48 = 16 \text{ tons.}$$

Dead panel load $= 2$ tons.

Live panel load $= 4$ tons.

By (3) we have lengths of 3–4, 5–6, and 7–8 = 4.375, 7.5, and 9.375 feet.

$$\text{Max. stress in } 2\text{–}3 = \frac{48 \times \sqrt{(10)^2 + (4.375)^2}}{10} = 52.4 \text{ tons.}$$

Since the dead load produces no stresses in the diagonals, the max. stress in any diagonal is found by putting only the live load on the bridge in the position to give the max. shear in that member (Art. 22).

Thus, for the maximum stress in 5–8 the live load is placed on the right, the center of moments is on the lower chord at 20 feet to the left of the point 2. The lever arm of 5–8 is 30 feet.

The reaction for this loading

$$= \tfrac{4}{8}(1 + 2 + 3 + 4 + 5) = 7.5 \text{ tons.}$$

$$\therefore \text{ Stress in } 5\text{–}8 = \frac{7.5 \times 20}{30} = 5 \text{ tons.}$$

For live load on the left, the stress in 5–8 = 0, and the counter 6–7 comes into action. The lever arm of 6–7 is 27.4 feet; the reaction for this loading = 6.5 tons. Therefore the stress S for 6–7 is found from the equation

$$6.5 \times 20 - 4(30 + 40) + S \times 27.4 = 0.$$

$$\therefore\ S = \text{stress in } 6\text{–}7 = 5\ 5 \text{ tons.}$$

If we cut a vertical as 5–6 by a section, place the live load on the right, and take the center of moments at the intersection of 3–5 and 4–6, which is 4 feet to the left of the point 2, we shall obtain the maximum *compression* in it caused by the live load. Thus,

The reaction for this loading = 7.5 tons.

∴ Max. stress in 5–6 due to live load

$$= -\frac{7.5 \times 4}{24} = -1.25 \text{ tons.}$$

Min. stress in 5–6 = 2.0 − 1.25 = 0.75 tons.

The maximum stress in each vertical is one of tension, and will occur when the live load covers the bridge, and is a dead and live panel load, or 6 tons.

If there be no live load on the truss, the stress in any vertical is a dead panel load, or a *tension* of 2 tons. Now let the live load come on from the right, a part of it which goes to the left support will cause *compression* in each vertical, or will diminish the tension in that vertical due to the dead load. Thus, when the live load reaches the joint 8′, it will cause in the vertical 9–10 a compression of 2.2 tons, which will neutralize the dead load tension of 2 tons, and leave as the result a compression of 0.2 tons. Also, when it reaches the apex 8, it will cause in the vertical 5–6 a compression of 1.25 tons, as we saw above, which will neutralize that much of the dead load tension of 2 tons, and leave as the result a tension of 0.75 tons.

The following stresses are found in a manner similar to the above:

CHORD STRESSES.

MEMBERS.	2–3	3–5	5–7	7–9	2–4, 4–6, etc.
Max. . . .	−52.4	−50.3	−48.8	−48.1	+48.0
Min. . . .	−17.5	−16.8	−16.3	−16.0	+16.0

STRESSES IN DIAGONALS.

MEMBERS.	3–6	5–8	7–10	4–5	6–7	8–9
Max. . . .	+4.3	+5.0	+5.5	+5.0	+5.5	+5.7
Min. . . .	0.0	0.0	0.0	0.0	0.0	0.0

STRESSES IN VERTICALS.

MEMBERS.	3–4	5–6	7–8	9–10
Max.	+6.0	+6.0	+6.0	+6.0
Min.	0.0	+0.8	+0.0	−0.2

Prob. 87. A parabolic bowstring as a through bridge, Fig. 44, has 8 panels, each 10 feet long and 10 feet center depth; the verticals are ties and the diagonals are braces; the dead and live loads are 400 lbs. and 1600 lbs. per foot per truss: find the stresses in all the members.

Ans. Max. stress in each panel of lower chord = +80 tons.

Max. stresses in upper chord
$= -87.3,\ -83.8,\ -81.4,\ -80.2$ tons.

Max. stress in 3–6
$= -8.7$, in 5–8 $= -10.0$, in 7–10 $= -10.9$, in 4–5
$= -10.0$, in 6–7 $= -10.9$, in 8–9 $= -11.3$ tons.

Max. stresses in verticals
$= +10.0,\ +12.5,\ +14.0,\ +14.5$ tons.

Min. stresses in verticals $= +2.0,\ +2.0,\ +2.0,\ +2.0$ tons.

Min. stresses in diagonals $= 0$.

Prob. 88. A through parabolic bowstring truss, Fig. 44, has 8 panels, each 12 feet long and 16 feet center depth; the verticals are ties and the diagonals are braces; the dead and live loads are 500 lbs. and 2000 lbs. per foot per truss: find the stresses in all the members.

Ans. Max. stress in each panel of lower chord = + 90 tons.

Min. stress in each panel of lower chord = + 18 tons.

Max. stresses in upper chord
$= -104.1,\ -97.5,\ -92.7,\ -90.3$ tons.

Min. stresses in upper chord
$= -20.8, -19.5, -18.5, -18.1$ tons.

Max. stresses in verticals
$= +15.0, +18.7, +21.0, +21.7$ tons.

Min. stresses in verticals $= +3.0, +3.0, +3.0, +3.0$ tons.

Max. stress in 3–6
$= -10.4$, in 5–8 $= -12.7$, in 7–10 $= -14.4$, in 4–5 $= -12.7$, in 6–7 $= -14.4$, in 8–9 $= -15.0$ tons.

Min. stresses in diagonals $= 0$.

Prob. 89. A through parabolic bowstring truss has 12 panels, each 8 feet long, and 12 feet center depth; the verticals are ties and the diagonals are braces; the dead and live loads are 500 lbs. and 2000 lbs. per foot per truss: find the stresses in all the members.

Art. 35. The Circular Bowstring Truss.—This form of truss, Fig. 45, is often used for highway bridges. The

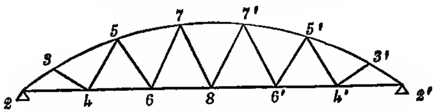


Fig. 45

joints of the upper chord lie upon the arc of a circle, each joint being directly over the middle of the panel below. The diagonals are built to take either tension or compression.

Prob. 90. A through circular bowstring truss, Fig. 45, has 6 panels, each 12 feet long, and 11.7 feet center depth; the joints of the upper chord lie on the arc of a circle of 60 feet radius; the dead and live loads are 450 lbs. and 1360 lbs. per foot per truss: find the stresses in all the members.

Ans. Max. stress in 2–3 = – 49.4, in 3–5 = – 54.3, in 5–7 = – 51.1, in 7–7′ = – 50.1 tons.

Max. stress in 2–4 = + 41.1, in 4–6 = + 45.8, in 6–8 = + 47.3 tons.

Max. stress in 3–4 = + 9.9, in 4–5 = + 9.6, in 5–6 = + 10.4, in 6–7 = + 10.0, in 7–8 = + 10.8 tons.

Min. stress in 4–5 = – 2.2, in 5–6 = – 1.3, in 6–7 = – 3.6, in 7–8 = – 3.1 tons.

This problem is taken, with slight changes, from Burr's Stresses in Bridge and Roof Trusses.

Prob. **91.** A through circular bowstring truss has 8 panels, each 15 feet long, the center depth is 19.74 feet; the joints of the upper chord lie on the arc of a circle of 100 feet radius, each joint being directly over the center of the panel below; the dead and live loads are 1000 lbs., and 2000 lbs. per foot per truss: find the stresses in all the members.

Art. 36. Snow Load Stresses.—For railway bridges the snow load is not taken into account, since the floor is open, so that but little is retained. For highway bridges the snow load is taken from 0 to 15 lbs. per square foot of floor surface, depending upon the climate where the bridge is situated.* The snow load is taken lower than for roofs, since the full live load is not likely to come upon the bridge while it is heavily loaded with snow. Since the snow load is uniform, the stresses due to it are computed in the same way as the dead load stresses; or the snow load and dead load stresses are proportional to the corresponding apex loads (Art. 8).

* In building highway bridges in England and France the snow load is not generally considered.

Prob. **92.** A through Howe truss, Fig. 32, has 10 panels, each 12 feet long and 12 feet deep; the width of roadway is 20 feet, and the width of each sidewalk is 5 feet; the snow load is 10 lbs. per square foot of floor: find the snow load stresses in all the members.

Ans. Stresses in lower chord
$= 4.1,\ 7.2,\ 9.5,\ 10.8,\ 11.3$ tons.
Stresses in the verticals $= 4.1,\ 3.2,\ 2.3,\ 1.4,\ 0.9$ tons.
Stresses in the diagonals
$= -5.7,\ -4.4,\ -3.2,\ -.9,\ -10.6$ tons.

Prob. **93.** A deck Pratt truss, Fig. 34, has 10 panels, each 16 feet long and 16 feet deep; the width of roadway, including sidewalks, is 35 feet; the snow load is 15 lbs. per square foot of floor: find the snow load stresses in all the members.

Art. 37. Stresses due to Wind Pressure.—In bridges of large span, it is often found that the stresses produced in some of the members by a gale of wind are almost as great as those caused by both the dead and live loads. In the principal members of the Forth bridge, Scotland, Sir Benjamin Baker estimated that the maximum stresses due to these three separate forces were as follows:

Stress due to dead load $= 2282$ tons.
Stress due to live load $= 1022$ tons.
Stress due to wind load $= 2920$ tons.

In estimating the wind pressure, and the resulting stresses in the members of a bridge, the practice of engineers varies greatly. Different railroads have their own specifications for wind pressure. The standard wind pressure per square foot ranges in this country from 30 to 50 lbs. It is assumed by many engineers that there will be no train on the bridge

when the wind blows with greater pressure than 30 lbs. per square foot. A wind pressure of 30 lbs. per square foot will overturn an empty freight car. In such a gale it would be hardly possible for a train to reach the bridge. It might, however, be caught there by a sudden squall; and this is just what appears to have happened in the case of the Tay bridge, which was destroyed by a wind storm in 1879. A maximum pressure of 50 lbs. per square foot is taken as applying to the bridge alone.

The surface exposed to the wind action is commonly taken as double the side elevation of one truss, for the reason that the windward truss cannot afford much shelter to the leeward truss, whatever may be the direction of the wind. In a heavy gale of wind there is not much shelter to be found under the lee of a lamp-post at a distance of 20 feet from it, even if the post be directly to windward. Experiments show that the wind pressure against the two trusses of a bridge is more than 1.8 times that on the exposed surface of one truss.

For Highway Bridges the wind pressure is frequently taken at about 30 lbs. per square foot of exposed surface of both trusses. It is assumed that there will be no live load upon the bridge when the wind is blowing at this maximum pressure of 30 lbs. per square foot.

For Railroad Bridges the wind pressure is taken at about 30 lbs. per square foot of exposed surface of both trusses, and about 300 lbs. per linear foot due to the train surface. The train surface is about 10 square feet for each linear foot of the bridge. The 30 lbs. per square foot of the trusses is treated usually as a dead load, and the 300 lbs. per linear foot due to the train surface as a live load. This regards each truss as fully exposed even when the train is on, though it partially shelters one truss.

To estimate the 30 lbs. pressure per square foot of exposed surface of both trusses, when this surface is not known, it is customary now to use the following rule: *Take 150 lbs. per linear foot per truss, or 75 lbs. per linear foot for each chord.*

The wind load on the trusses is assumed to be divided equally between the upper and lower lateral trusses, while the wind load on the train is all taken by the lateral truss belonging to the loaded chord. The lateral trusses are the horizontal trusses placed between the chords of the vertical trusses. The chords of the vertical trusses are also the chords of the lateral trusses. The lateral trusses of a bridge are either of the Pratt, Howe, or Warren type, corresponding generally with the type of the main trusses.

REM. — In finding wind stresses the loads on the lateral system of the unloaded chord are considered as applied equally upon the two sides, windward and leeward, while the loads on the lateral system of the loaded chord are considered as applied wholly on the windward side. The stresses are then readily found by methods already familiar for finding dead load and live load stresses.

The lower lateral system in a through bridge and the upper lateral system in a deck bridge are calculated for a dead load of 30 lbs. per square foot of exposed surface of both trusses, and a live load of 300 lbs. per linear foot of train; while the other lateral system is calculated only for a dead load of 30 lbs. per square foot of exposed surface of both trusses.

The resulting chord stresses should be combined with those due to the dead load, or with those due to the dead and live loads if the live load acts with the wind load. The lower chord is always in tension under the action of the dead and live loads. When the wind acts the bridge is bent laterally, and the windward lower chord has its maximum tension diminished by the compression due to the wind,

while the lower leeward chord has its tension increased by the tension due to the wind. The compression in the windward chord due to the wind may exceed the tension due to the dead load, or to the dead and live loads combined. The lower chord on each side then should not only be able to sustain the total maximum of dead, live, and wind loads, but to act as a strut to resist compression.

Prob. 94. A through Pratt truss railway bridge 16 feet wide, half of which is shown in Fig. 47, has 12 panels, each 16 feet long; the upper and lower lateral systems, Figs. 46

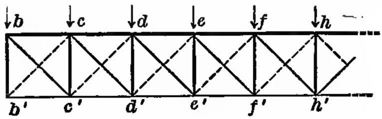


Fig. 46

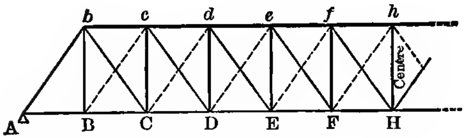


Fig. 47

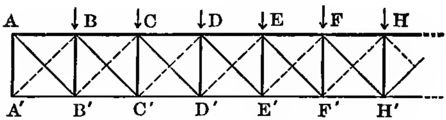
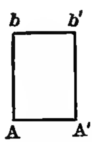


Fig. 48



and 48, are Pratt trusses: find the stresses in both lateral systems due to a wind pressure of 30 lbs. per square foot of exposed surface of both trusses and 300 lbs. per linear foot of train surface.

(1) Upper Lateral System (10 Panels).

Panel load for one chord (see rule) $= \frac{75 \times 16}{2000} = 0.6$ tons.

This load acts as a dead load at each apex of the windward and leeward chords in the direction of the arrows, shown in Fig. 46; hence there is no stress in the dotted diagonals.

$$\tan\theta = 1; \quad \sec\theta = 1.414.$$

Stress in $hh' = -0.6$ tons.
Stress in $ff' = -0.6 \times 2 = -1.2$ tons.
Stress in $ee' = -0.6 \times 4 = -2.4$ tons.
Stress in $bc = -0.6 \times 9 \times 1 = -5.4$ tons.
Stress in $cd = -[5.4 + 4.2] \times 1 = -9.6$ tons.
Stress in $fh' = +0.6 \times 1.414 = +0.8$ tons.
Stress in $ef' = +3 \times 0.6 \times 1.414 = +2.5$ tons.

(2) Lower Lateral System (12 Panels).

Panel load for both chords

$$= \frac{150 \times 16}{2000} + \frac{300 \times 16}{2000} = 1.2 \text{ tons} + 2.4 \text{ tons}.$$

The former we treat as a dead load, the latter as a live load, both acting at each apex of the windward chord in the direction of the arrows, shown in Fig. 48. (See Remark.)

Stress in $AB = -5.5 \times 3.6 = -19.8$ tons.

Stress in CC'

$$= -\left[4.5 \times 1.2 + \frac{2.4}{12}(1 + 2 + 3 + \cdots + 10)\right] = -16.4 \text{ tons}.$$

Stress in DE'

$$= [2.5 \times 1.2 + 0.2(1 + 2 + 3 + \cdots + 8)]1.414 = 14.4 \text{ tons}.$$

Thus the following stresses are computed:

UPPER LATERAL SYSTEM.

Stresses in chords
$= -5.4, -9.6, -12.6, -14.4, -15.0$ tons.

Stresses in struts
$= -5.7, -4.8, -3.6, -2.4, -1.2, -0.6$ tons.

Stresses in diagonals
$= +7.6, +5.9, +4.2, +2.5, +0.8$ tons.

LOWER LATERAL SYSTEM.

Stresses in chords $= 19.8, 36.0, 48.6, 57.6, 63.0, 64.8$ tons.

Stresses in struts
$= -21.6, -19.8, -16.4, -13.2, -10.2, -7.4, -5.4$ tons.

Stresses in diagonals $= 27.9, 23.1, 18.6, 14.4, 10.4, 6.8$ tons.

The chord *bcdh* is in compression under the action of the dead and live loads; this compression is increased by that due to the wind pressure, as just found.

The chord $A'B'C'H'$ is in tension under the action of the dead and live loads; this tension is increased by that due to the wind pressure, as just found.

When the wind blows on the opposite side of the bridge the diagonal and chord stresses are to be interchanged, while the strut stresses remain the same. Thus there will be the same stresses in $b'c'$, $c'd'$, etc., as in *bc*, *cd*, etc., and the same in $A'B'$, $B'C'$, etc., as in *AB*, *BC*, etc., and the dotted system of braces will act in each lateral.

The wind loads on the left half of the upper lateral are transferred to the abutment at *A* by means of the *portal bracing* $AA'b'b$ in the transverse plane of *Ab*.

Prob. 95. A through Howe truss railway bridge 20 feet wide has 12 panels, each 20 feet long; the upper and lower lateral systems are Howe trusses: find the stresses in both lateral systems due to the same wind pressure as in the previous problem.

By taking the dotted diagonals, Figs. 47, 46, and 48 will represent the left half of the Howe truss and the upper and lower lateral systems. Then by *rule:*

Panel load for one chord of upper lateral = 0.75 tons, to be taken as a dead load.

Panel load for both chords of lower lateral

$$= 1.5 \text{ tons} + 3.0 \text{ tons};$$

the former to be taken as a dead load, the latter as a live load, all acting in the direction of the arrows.

Stress in $b'c = -9 \times .75 \times 1.414 = -9.5$ tons.

Stress in $C'D$

$$= -[3.5 \times 1.5 + \tfrac{3}{12}(1 + 2 + \cdots + 9)]1.414 = -23.3 \text{ tons.}$$

Stress in $B'C' = 10 \times 4.5 \times 1 = 45$ tons.

Upper Lateral System.

Stresses in chords = 6.8, 12.0, 15.8, 18.0, 18.8 tons.

Stresses in struts = 0.4, 6.0, 4.5, 3.0, 1.5, 0.8 tons.

Stresses in diagonals

= − 9.5, − 7.4, − 5.3, − 3.2, − 1.1 tons.

Lower Lateral System.

Stresses in chords = 24.8, 45.0, 60.8, 72.0, 78.8, 81.0 tons.

Stresses in struts = 2.3, 20.5, 16.5, 12.8, 9.2, 6.0, 3.8 tons.

Stresses in diagonals

= − 34.9, − 28.9, − 23.3, − 18.0, − 13.0, − 8.5 tons.

Prob. 96. A deck Pratt truss railroad bridge 20 feet wide has 10 panels, each 20 feet long; the upper and lower lateral systems are Pratt trusses: find the stresses in both lateral systems due to a wind pressure of 40 lbs. per square foot of exposed surface of both trusses and 300 lbs. per linear foot of train surface.

By taking only the full diagonals, Fig. 46 will represent the left half of the deck Pratt truss and also the upper and lower lateral systems.

Since this is a deck bridge, the upper laterals are on the loaded, and the lower laterals are on the unloaded chord. Hence by the *rule:*

Panel load for one chord on lower lateral system

$$= \frac{100 \times 20}{2000} = 1 \text{ ton},$$

to be taken as a dead load at each apex of the windward and leeward chords (Rem.).

Panel load for both chords on upper lateral system

$$= \frac{200 \times 20}{2000} + \frac{300 \times 20}{2000} = 2 \text{ tons} + 3 \text{ tons};$$

the former to be taken as a dead load, the latter as a live load, both acting at each apex of the windward chord (Rem.), and all acting in the direction of the arrows in Fig. 46.

Upper Lateral System.

Stresses in chords

$= -22.5,\ -40.0,\ -52.5,\ -60.0,\ -62.5$ tons.

Stresses in struts

$= -25.0,\ -22.5,\ -17.8,\ -13.4,\ -9.3,\ -5.0$ tons.

Stresses in diagonals

$= +31.7,\ +25.1,\ +18.9,\ +13.1,\ +7.7$ tons.

Lower Lateral System.

Stresses in chords
$= -9.0, -16.0, -21.0, -24.0, -25.0$ tons.

Stresses in struts
$= -9.5, -8.0, -6.0, -4.0, -2.0, -1.0$ tons.

Stresses in diagonals
$= +12.7, +9.9, +7.1, +4.2, +1.4$ tons.

Prob. 97. A through Pratt truss railroad bridge $16\frac{1}{4}$ feet wide has 9 panels, each 17 feet long; the upper and lower lateral systems are Pratt trusses: find the stresses in both lateral systems due to the same wind pressures as in Prob. 94.

Art. 38. The Factor of Safety for a body is the ratio of its breaking to its working stress; or it is the ratio of the load which will just crush the body to the assumed load, or load which it is intended to carry. Thus, if the tearing unit-stress of an iron plate be 20 tons and the working unit-stress be 4 tons, the factor of safety will be 5.

The value of the factor of safety is generally assumed by the engineer; different engineers assume different factors of safety, depending somewhat upon the manner of applying the loads, the character of the structure, and the nature and quality of the material. Thus, for steady loads and slowly varying stresses, the factor of safety may be low; but when the load is applied with shocks and sudden stresses, the factor ought to be large. In a building the stresses on the walls are steady, and hence the factor of safety may be low. In a bridge the stresses on the different members are more or less varying, and hence the factor of safety must be higher.

Also, it has been seen (Art. 16) that the live load for a short span is much more than that for a long span. For this reason the variation of stress in passing from a loaded to an unloaded state is much greater in the members of a short span than in those of a long one. Consequently, the material in a short span will suffer what is termed *fatigue* more than in a long one. And although the fatigue of metals is a subject not yet well understood, yet it is clearly established that these sudden changes of stress in short spans demand a larger factor of safety than those in long spans.

In American practice the values of the factor of safety, for steady and for varying stresses, are: from 3 to 7 for wrought iron, from 5 to 15 for cast iron, from 4 to 8 for steel, from 8 to 12 for timber, and from 12 to 30 for stone or brick.

At times it may be necessary to employ a much larger factor of safety than either of these, owing to local circumstances. If the risk attending failure (such as loss of life or property) is small, the factor of safety may be small. But if the risk is large, the factor of safety must be large in proportion. With a bridge in perfect condition, a very small factor would be sufficient. But no bridge is in perfect condition; all rough places, such as rail joints, more or less open, produce shocks which cause sudden stresses. These stresses cannot be measured. A very large allowance has to be made for these uncertainties and for the imperfect state of our knowledge; and therefore there must be a large factor of safety to cover all uncertainties.

CHAPTER III.

BRIDGE TRUSSES WITH UNEQUAL DISTRIBUTION OF THE LOADS.

Art. 39. Preliminary Statement. — The preceding chapter has treated of stresses produced by dead loads and uniformly distributed live loads. While this is the general method of treatment for *highway* bridges, and in English practice for *railway* bridges, it has become the general practice of American engineers to calculate these stresses for railway bridges by one of the following three methods:

(1) The use of a *uniformly distributed excess load* covering one or more panels followed by a uniform train load covering the whole span.

(2) The use of one or two *concentrated excess loads* with a uniform train load covering the span.

(3) The use of the *actual specified locomotive wheel loads* followed by a uniform train load.

It is proposed in this chapter to show how to find the maximum stress in each member of a truss by each of these three methods.

Art. 40. Method of Calculating Stresses when the Uniform Train Load is preceded by one or more Heavy Excess Panel Loads. — This method of finding maximum stresses is sometimes used to avoid the laborious practice of finding the stresses due to the actual locomotive wheel loads to be described later.

Prob. **98.** A through double Warren truss, Fig. 49, has 10 panels, each 12 feet long and 12 feet deep; the dead load is 1000 lbs. per foot per truss, and the train load is

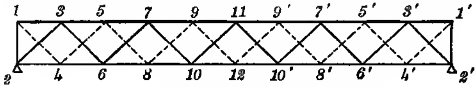


Fig. 49

2000 lbs. per foot per truss, preceded by one locomotive panel load of 30 tons per truss: find the stresses in all the members.

We may first find the stresses caused by the uniform dead and train loads, and then the stresses caused by the excess locomotive load, and add the results; or, we may determine the maximum stress in each member directly by one equation, as we have generally done; and this is the simplest method.

Dead panel load per truss = 6 tons.

Train panel load per truss = 12 tons.

Locomotive panel load per truss = 30 tons.

Locomotive excess panel load per truss = 18 tons.

$$\tan\theta = 1;\ \sec\theta = 1.414.$$

For the maximum chord stresses (except 2–4) the locomotive must stand at 4, and the train panel loads at all the other joints; for the maximum stress in 2–4 the locomotive must be put at 6, as can easily be seen.* Then,

Max. stress in 2–4

$$= 2 \times 6 + \tfrac{8}{10} \times 30 + \tfrac{12}{10}(2 + 4 + 6) = 50.4 \text{ tons}.$$

Max. stress in 4–6

$$= 2 \times 18 + 4 \times 18 + (\tfrac{9}{10} - \tfrac{1}{10})18 = 122.4 \text{ tons}.$$

* This method does not give strictly the maximum stresses in all the chord members. While the error near the ends of the truss is quite small, it increases towards the middle.

The minimum chord stresses are due to dead load alone.

For the maximum stress in any diagonal the live load is placed on the right of a section cutting that diagonal, as by the usual method. Thus,

Max. stress in 5–8

$$= [1\tfrac{1}{2} \times 6 + 30 \times \tfrac{7}{10} + 1.2(1+3+5)]1.414 = 57.7 \text{ tons.}$$

Min. stress in 7–10

$$= [1 \times 6 - 30 \times \tfrac{2}{10}]1.414 = 0.0.$$

That is, the dead load passing through 7–10 to the left abutment is neutralized by the part of the locomotive load passing through 7–10 to the right abutment.

The following stresses are found in a manner similar to the above:

UPPER CHORD STRESSES.

MEMBERS.	1–3	3–5	5–7	7–9	9–11
Max.	−61.2	−133.2	−183.6	−219.6	−234.0
Min.	−15.0	− 39.0	− 57.0	− 69.0	− 75.0

LOWER CHORD STRESSES.

MEMBERS.	2–4	4–6	6–8	8–10	10–12
Max.	+50.4	+122.4	+176.4	+208.8	+226.8
Min.	+12.0	+ 36.0	+ 54.0	+ 66.0	+ 72.0

DIAGONAL STRESSES.

MEMBERS.	1–4	3–6	5–8	7–10	9–12	11–10′
Max.	+86.5	+71.3	+57.7	+44.1	+32.2	+20.3
Min.	+21.2	+17.0	+ 8.5	+ 0.0	−10.3	−20.3

Max. stress in 1–2 = − 61.2 tons.

Prob. 99. A deck Pratt truss, Fig. 50, has 10 panels, each $12\frac{1}{2}$ feet long and $12\frac{1}{2}$ feet deep; the dead load is 800 lbs.

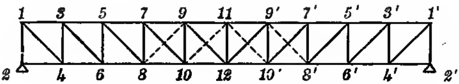


Fig. 50

per foot per truss, and the train load is 1600 lbs. per foot per truss preceded by two locomotive panel loads of 30 tons each per truss: find the stresses in all the members.

Consider three fifths of the dead load as applied at the upper chord, and two fifths at the lower chord.*

Dead panel load per truss $= 5$ tons.

Train panel load per truss $= 10$ tons.

Excess panel load per truss $= 20$ tons.

For maximum chord stresses put locomotive panel loads at joints 3 and 5, and the train panel loads at all the other joints. Thus,

Max. stress in 1–3

$$= 4\tfrac{1}{2} \times 15 + 20\left(\tfrac{9}{10} + \tfrac{8}{10}\right) = 101.5 \text{ tons} = \text{stress in 4–6}.$$

Max. stress in 3–5

$$= 8 \times 15 + 20\left(\frac{9}{10} + \frac{8}{10} + \frac{8-1}{10}\right) = 168.0 \text{ tons}.$$

Max. stress in 5–7

$$= 10\tfrac{1}{2} \times 15 + \tfrac{20}{10}(17 + 7 - 3) = 199.5 \text{ tons}.$$

For maximum stress in any diagonal the live load is placed on the right, by the usual method.

* In railroad bridges it is customary to take two thirds of the dead load as applied at the loaded chord; that is, the chord which carries the live load, and one third at the unloaded chord.

CHORD STRESSES.

MEMBERS.	1-3	3-5	5-7	7-9	9-11
Max.	−101.5	−168.0	−199.5	−216.0	−217.5
Min.	− 22.5	− 40.0	− 52.5	− 60.0	− 62.5

STRESSES IN THE DIAGONALS.

MEMBERS.	1-4	3-6	5-8	7-10	9-12	8-9	10-11
Max. . .	+143.5	+118.0	+94.0	+71.3	+50.2	+12.1	+30.4
Min. . .	+ 31.7	+ 20.4	+ 4.9	0.0	0.0	0.0	0.0

Max. compression in the posts = 102.0, 99.5, 81.5, 64.5, 48.5, 36.0 tons.

Prob. 100. A through Howe truss, Fig. 32, has 10 panels, each 12 feet long and 12 feet deep; the dead load is 1000 lbs. per foot per truss, and the train load is 1667 lbs. per foot per truss, preceded by two locomotive panel loads of 30 tons each per truss: find the stresses in all the members.

Dead panel load = 6 tons.
Train panel load = 10 tons.
Excess panel load = 20 tons.

Consider one third of the dead load as applied at the upper chord.

CHORD STRESSES.

MEMBERS.	2-4	4-6	6-8	8-10	10-12
Max.	+106.0	+176.0	+210.0	+228.0	+230.0
Min.	+ 27.0	+ 48.0	+ 63.0	+ 72.0	+ 75.0

STRESSES IN THE DIAGONALS.

MEMBERS.	2–3	4–5	6–7	8–9	10–11	7–10	9–12
Max. . .	−149.9	−123.0	−97.6	−73.6	−50.9	−9.9	−29.7
Min. . .	− 38.0	− 25.5	− 8.5	0.0	0.0	0.0	0.0

STRESSES IN THE VERTICALS.

MEMBERS.	3–4	5–6	7–8	9–10	11–12
Max.	+104.0	+85.0	+67.0	+50.0	+37.0
Min.	+ 25.0	+16.0	+ 4.0	+ 4.0	+ 4.0

Prob. **101.** A through Pratt truss, Fig. 33, has 10 panels, each 15 feet long and 15 feet deep; the dead load is 1200 lbs. per foot per truss, and the train load is 2000 lbs. per foot per truss, preceded by two locomotive panel loads of 30 tons each per truss: find the stresses in all the members.

Consider one third of the dead load as applied at the upper chord.

CHORD STRESSES.

MEMBERS.	2–4	4–6	6–8	8–10	10–12	9–11
Max. . .	+133.5	+133.5	+228.0	+283.5	+315.0	−322.5
Min. . .	+ 40.5	+ 40.5	+ 72.0	+ 94.5	+108.0	−112.5

STRESSES IN THE DIAGONALS.

MEMBERS.	2–3	3–6	5–8	7–10	9–12	8–9	10–11
Max.	−188.8	+152.7	+118.7	+87.0	+57.3	+4.2	+29.7
Min.	− 57.3	+ 40.3	+ 19.1	0.0	0.0	0.0	0.0

STRESSES IN THE VERTICALS.

MEMBERS.	3-4	5-6	7-8	9-10	11-12
Max.	+39.0	−87.0	−64.5	−43.5	−28.5
Min.	+ 9.0	−16.5	− 3.0	− 3.0	− 3.0

Prob. 102. A through Whipple truss, Fig. 37, has 12 panels, each 12 feet long and 24 feet deep; the dead load per foot per truss is 1000 lbs., and the train load is 2000 lbs. per foot per truss, preceded by one locomotive panel load of 30 tons per truss: find the stresses in all the members.

For max. chord stresses put locomotive panel load at joint 4 and the train panel loads at all the other joints. *

$$\text{Dead panel load} = 6 \text{ tons.}$$

$$\text{Train panel load} = 12 \text{ tons.}$$

$$\text{Excess panel load} = 18 \text{ tons.}$$

$$\text{Max. stress in 4–6} = (3 \times 18 + \tfrac{11}{12} \times 18) \times .5$$
$$= 35.3 \text{ tons.}$$

$$\text{Max. stress in 6–8} = 35.3 + 2\tfrac{1}{2} \times 18 = 80.3 \text{ tons.}$$

$$\text{Max. stress in 8–10} = 80.3 + 2 \times 18 - \tfrac{1}{12} \times 18$$
$$= 114.8 \text{ tons.}$$

CHORD STRESSES.

MEMBERS.	2-4	4-6	6-8	8-10	10-12	12-14	9-11
Max. . . .	0.0	+35.3	+80.3	+114.8	+141.8	+158.3	−167.3
Min. . . .	0.0	+ 9.0	+24.0	+ 36.0	+ 45.0	+ 51.0	− 54.0

* Put all the dead load on the loaded chord, unless otherwise stated.

STRESSES IN THE DIAGONALS.

MEMBERS.	1–4	1–6	3–8	5–10	7–12	9–14	11–12′	13–10	11–8
Max. . .	+78.8	+84.8	+71.4	+59.0	+46.0	+33.9	+23.3	+12.7	+3.5
Min. . .	+20.1	+21.2	+13.4	+ 5.7	0.0	0.0	0.0	0.0	0.0

STRESSES IN THE VERTICALS.

MEMBERS.	1–2	3–4	5–6	7–8	9–10	11–12	13–14
Max. . .	−115.5	−50.5	−41.0	−32.5	−24.0	−16.5	−9.0
Min. . .	− 33.0	− 9.5	− 4.0	0.0	0.0	0.0	0.0

Prob. 103. A through Whipple truss, Fig. 38, has 16 panels, each 10 feet long and 20 feet deep; the dead and train loads are 1000 lbs. and 3000 lbs. per foot per truss, the train being preceded by one locomotive panel load of 30 tons per truss: find the stresses in all the members.

Art. 41. Method of Calculating Stresses when one Concentrated Excess Load accompanies a Uniform Train Load.—This method is sometimes used, like the one in Art. 40, to avoid the practice of finding the stresses due to the locomotive wheel loads. But we have seen that the method in Art. 40 does not give strictly the maximum stresses in all the chord members.

Let Fig. 51 be a truss supporting a uniform train load covering the span, and a concentrated excess load P. We suppose the excess load P to be the difference between the locomotive panel load and the uniform train panel load as in Art. 40. Then the stress in any chord member, as bd, caused by the concentrated load P, is equal to the bending moment at c, the center of moments for bd, divided by the lever arm for the chord; and hence the chord stress will be

a maximum when the concentrated load P is so placed as to make the bending moment a maximum. Now, for a single concentrated load, the maximum bending moment at any

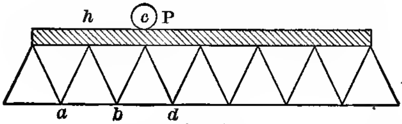


Fig. 51

point occurs when the load is at that point; for, if the load be moved to either side of the point, the reaction of the opposite abutment will be diminished, and hence the moment will be diminished.

Therefore, for a concentrated excess load and a uniform train load, the maximum bending moment at any point, and consequently the maximum chord stress in any member, occurs when the concentrated load is at the center of moments for that member, or at the vertical section through the center of moments, and the uniform train load covers the whole span (Art. 22).

Thus, for the maximum stress in *bd*, the concentrated load P is at *c*, the center of moments for *bd*; and so for any other panel in the lower chord, or unloaded chord generally. For any panel in the upper, or loaded chord, as *hc*, of a truss like the *Warren*, the concentrated load P acts at *c*, or at that end of the panel which is to the right of the vertical section through *b*, the center of moments for *hc*, while the uniform train load covers the whole span (Art. 22). For any panel in the *loaded chord* of a truss like the *Pratt* or *Howe*, where the apexes of the upper chord are vertically above those of the lower chord, the concentrated load P is at the apex directly over or under the center of moments for that member.

For a single concentrated load, the maximum positive shear at any section will occur when the load is just to the right of the section; for the left reaction is then a maximum.

Therefore, for a concentrated excess load and a uniform train load, the maximum stress in any brace occurs when the concentrated load is at the panel point immediately on the right of the section, and the uniform train load covers the span from the right abutment to this same panel point.

Thus, the greatest positive shear, and therefore the maximum stress, in *bc* or in *bh*, occurs when the concentrated load is at the apex *c* and the uniform train load extends from this apex to the right abutment. The greatest negative shear for *bc* or for *bh* would occur when the concentrated load is at the apex *h* and the uniform train load reaches from this apex to the left abutment. For maximum chord stresses, cars both precede and follow the locomotive. This does not often happen. For maximum stresses in the braces the locomotive precedes the cars. It follows therefore, that maximum chord stresses are of less frequent occurrence than maximum web stresses, which occur for every passage of the train.

Prob. **104.** A through Howe truss, Fig. 32, has 10 panels, each 12 feet long and 12 feet deep; the dead and train loads are 1000 lbs. and 2000 lbs. per foot per truss, and the locomotive panel load is 30 tons per truss: find the stresses in all the members.

Consider one third of the dead load as applied at the upper chord.

Dead panel load $= 6$ tons.

Train panel load $= 12$ tons.

Excess panel load $= 18$ tons.

For the maximum stress in any chord member, as 6–8, the excess load is placed at the joint 8 and the train load covers the whole span. Then, (2) of Art. 19,

Max. stress in

$$6\text{–}8 = 10.5 \times 18 + \tfrac{7}{10} \times 18 \times 3 = 226.8 \text{ tons.}$$

Otherwise by moments, (1) of Art. 19, thus:

Left reaction $= 4\frac{1}{2} \times 18 + \frac{7}{10} \times 18 = 93.6$ tons.

The equation of moments about the point 7 is

$$93.6 \times 36 - 18(12 + 24) - \text{stress in } 6\text{–}8 \times 12 = 0.$$

$\therefore$ max. stress in

$$6\text{–}8 = 93.6 \times 3 - 18 \times 3 = 226.8 \text{ tons, as before.}$$

For the maximum stress in any diagonal, as 4–5, the excess load is placed at 6, and the train loads cover the span from this point to the right abutment. Then,

Max. stress in

$$4\text{–}5 = -[3\tfrac{1}{2} \times 6 + 1.2(1 + 2 + \cdots + 8) + .8 \times 18]1.414$$
$$= -111.1 \text{ tons.}$$

Min. stress in

$$6\text{–}7 = -[2\tfrac{1}{2} \times 6 - \tfrac{12}{10} \times 3 - \tfrac{2}{10} \times 18]1.414 = -11 \text{ tons.}$$

CHORD STRESSES.

MEMBERS.	2–4	4–6	6–8	8–10	10–12
Max.	+ 97.2	+ 172.8	+ 226.8	+ 259.2	+ 270.0
Min.	+ 27.0	+ 48.0	+ 63.0	+ 72.0	+ 75.0

STRESSES IN THE DIAGONALS.

MEMBERS.	2–3	4–5	6–7	8–9	10–11	7–10	9–12
Max. . .	−139.7	−111.1	−86.5	−63.6	−42.4	−5.1	−22.9
Min. . .	− 38.0	− 25.4	−11.0	0.0	0.0	0.0	0.0

STRESSES IN THE VERTICALS.

MEMBERS.	3–4	5–6	7–8	9–10	11–12
Max.	+95.2	+76.6	+59.2	+43.0	+34.0
Min.	+25.0	+16.0	+ 5.8	+ 4.0	+ 4.0

Prob. 105. A through Pratt truss, Fig. 33, has 10 panels, each 15 feet long and 15 feet deep; the dead and train loads are 1200 lbs. and 2000 lbs. per foot per truss, and the excess load is 20 tons per truss: find the stresses in all the members.

Consider one third of the dead load as applied at the upper chord.

Dead panel load = 9 tons.
Train panel load = 15 tons.
Excess panel load = 20 tons.

Max. stress in 8–10 $= 10\frac{1}{2} \times 24 + \frac{7}{10} \times 20 \times 3 = 294$ tons.

Max. stress in

$$3\text{–}6 = [3\tfrac{1}{2} \times 9 + \tfrac{15}{10} \times 36 + \tfrac{20}{10} \times 8]\,1.414 = 143.5 \text{ tons.}$$

Max. stress in

$$7\text{–}8 = -[1\tfrac{1}{2} \times 9 + 3 + 1.5 \times 21 + 2 \times 6] = -60 \text{ tons.}$$

CHORD STRESSES.

MEMBERS.	2–4	4–6	6–8	8–10	10–12	9–11
Max. . .	+126.0	+126.0	+224.0	+294.0	+336.0	−350.0
Min. . .	+ 40.5	+ 40.5	+ 72.0	+ 94.5	+108.0	−112.5

STRESSES IN THE DIAGONALS.

MEMBERS.	2–3	3–6	5–8	7–10	9–12	8–9	10–11
Max. .	−178.2	+143.5	+111.0	+80.6	+52.3	+2.1	+26.2
Min. .	− 40.5	+ 39.6	+ 19.8	0.0	0.0	0.0	0.0

STRESSES IN THE VERTICALS.

MEMBERS.	3–4	5–6	7–8	9–10	11–12
Max.	41.0	−81.5	−60.0	−40.0	−26.0
Min.	+ 6.0	−17.0	− 3.0	− 3.0	− 3.0

Prob. 106. A double Warren truss, Fig. 49, used as a deck bridge, has 10 panels, each 14 feet long and 14 feet deep; the dead and train loads are 1000 lbs. and 2000 lbs. per foot per truss, and the excess load is 20 tons per truss: find the stresses in all the members.

UPPER CHORD STRESSES.

MEMBERS.	1–3	3–5	5–7	7–9	9–11
Max.	−60.0	−154.0	−213.0	−251.0	−268.0
Min.	−14.0	− 42.0	− 63.0	− 77.0	− 84.0

LOWER CHORD STRESSES.

MEMBERS.	2–4	4–6	6–8	8–10	10–12
Max.	+70.5	+152.5	+213.5	+253.5	+272.5
Min.	+17.5	+ 45.5	+ 66.5	+ 80.5	+ 87.5

DIAGONAL STRESSES.

MEMBERS.	1–4	3–6	5–8	7–10	9–12	11–10′
Max.	+82.0	+66.3	+50.6	+36.9	+23.2	+11.5
Min.	+19.8	+10.0	+ 0.3	−11.5	−23.2	−36.9

Max. stress in 1–2 $= 2\frac{1}{2} \times 21 + 10 = 62.5$ tons.

Prob. 107. A deck Pratt truss, Fig. 50, has 10 panels, each 15 feet long and 15 feet deep; the dead and train loads are 800 lbs. and 2000 lbs. per foot per truss, and the excess load is 20 tons per truss: find the stresses in all the members.

Art. 42. Method of Calculating Stresses when two equal Concentrated Excess Loads, placed about 50 feet apart, accompany a Uniform Train Load. — This method is often used, like the two in Arts. 40 and 41, to avoid the practice of finding the stresses due to the locomotive wheel loads, and is a nearer approximation to the actual loads than either of the other two.

Let Fig. 52 be a truss supporting a uniform train load covering the span, and two equal concentrated excess loads,

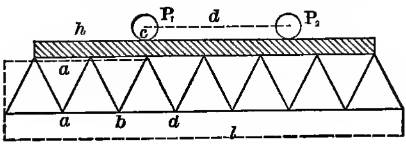


Fig. 52

P_1 and P_2. As in Arts. 40 and 41, we suppose each excess load to be the difference between the locomotive panel load and the uniform train panel load. Then the stress in any chord member, as *bd*, caused by the concentrated loads P_1 and P_2 is equal to the bending moment at *c*, the center of moments for *bd*, divided by the lever arm of *bd*; and hence the chord stress will be a maximum when the two concentrated loads are so placed as to make the bending moment a maximum.

Now, for two equal loads, a fixed distance apart, the maximum moment at any point occurs when one load is at

the point and the other is on the longer segment of the truss. This may be shown as follows:

Let l = length of span in Fig. 52, d = distance between the two equal loads P_1 and P_2, and a = distance from left abutment to center of moments c; then supposing the load P_1 to be at a distance x to the left of c, and calling M the moment at the point c, due to the two loads, P_1 and P_2, and denoting each load by P, since they are equal, we have

$$M = P\frac{a(2\,l - 2\,a - d) - (l - 2)x}{l}$$

$$= P\frac{a(2\,l - 2\,a - d)}{l}, \text{ a maximum, when } x = 0.$$

Similarly, if the load P_1 be placed at a distance x to the right of c, the moment at the point c will be a maximum when $x = 0$.

Therefore, for two equal concentrated excess loads, and a uniform train load, the maximum bending moment at any point, and consequently the maximum chord stress in any member, occurs when one of the equal concentrated loads is at the center of moments for that member, and the other concentrated load is on the longer segment of the truss, while the uniform train load covers the whole span.

Thus, for the maximum stress in bd, the concentrated load P_1 is at c, and the other load P_2 is to the right; and so for any other panel in the lower, or unloaded, chord of the left half of the truss. For any panel in the upper, or *loaded*, chord of the left half, as hc, of a truss like the *Warren*, where all the members are inclined, the load P_1 acts at c, or at that end of the panel which is to the right of the

vertical section through b, the center of moments for hc, and the other load P_2 is to the right.

For any panel in the *loaded chord* of a truss like the *Pratt* or *Howe*, where the upper apexes are vertically above the corresponding lower ones, the concentrated load P_1 is at the apex directly over or under the center of moments for that member.

For two equal concentrated loads P_1 and P_2, the maximum positive shear at any section occurs when both loads are on the right of the section and P_1 is as near to it as possible; for the left reaction is then a maximum.

Therefore, for two equal concentrated excess loads and a uniform train load, the maximum stress in any brace occurs when both loads are on the right of the section and one of them is at the panel point immediately on the right of the section, and the uniform train load covers the span from this point to the right abutment.

The second excess load P_2 should be placed at a panel point at an interval of *about* 50 feet to the right of P_1, to simplify the computation. Thus, should the panel length be 12, 15, or 18 feet, the distance between the loads would be 48, 45, or 54 feet.

Prob. **108.** A through Warren truss, Fig. 53, has 8 panels, each 10 feet long and 10 feet deep, its web members all

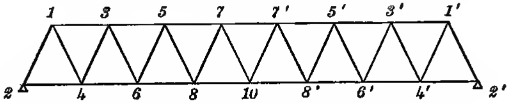


Fig. 53

forming isosceles triangles; the dead and train loads are 1000 lbs. and 2000 lbs. per foot per truss, and there are two

excess loads 50 feet apart, each of 33 tons: find the stresses in all the members.

$$\begin{aligned} \text{Dead panel load} &= 5 \text{ tons.} \\ \text{Train panel load} &= 10 \text{ tons.} \\ \text{Each excess load} &= 33 \text{ tons.} \\ \tan\theta = \tfrac{1}{2};\ \sec\theta &= 1.117. \end{aligned}$$

For the maximum stress in any chord member, as 4–6, one excess load is placed at 6 and the other one at 4′, or 50 feet to the right of the first, and the dead and train loads cover the whole span. Then (2) of Art. 19,

$$\text{Max. stress in } 4\text{–}6 = [9\tfrac{1}{2} \times 15 + \tfrac{33}{8}(1 + 6)3] \times .5$$
$$= 114.6 \text{ tons.}$$

Otherwise by moments, (1) of Art. 19, as follows:

$$\text{Left reaction} = 3\tfrac{1}{2} \times 15 + 33(\tfrac{1}{8} + \tfrac{6}{8}) = 81.375 \text{ tons.}$$

The equation of moments about the point 3 is

$$81.375 \times 15 - 15 \times 5 - \text{stress in } 4\text{–}6 \times 10 = 0.$$

$$\therefore \text{ max. stress in } 4\text{–}6 = 114.6 \text{ tons, as before.}$$

For the maximum stress in any diagonal, as 3–6, one excess load is placed at 6 and the other 50 feet to the right of it at 4′, and the train loads cover the span from 6 to the right abutment. Then,

$$\text{Max. stress in } 3\text{–}6 = [2\tfrac{1}{2} \times 5 + \tfrac{10}{8}(1 + 2 + \cdots + 6)$$
$$+ \tfrac{33}{8}(6 + 1)]1.117 = 75.5 \text{ tons} = -\text{stress in } 3\text{–}4.$$

$$\text{Min. stress in } 5\text{–}6 = -[1\tfrac{1}{2} \times 5 - \tfrac{3}{8} \times 10 - \tfrac{2}{8} \times 33]1.117.$$
$$= 5.0 \text{ tons.}$$

UPPER CHORD STRESSES.

MEMBERS.	1–3	3–5	5–7	7–7′
Max.	−89.6	−147.8	−174.4	−186.0
Min.	−17.5	− 30.0	− 37.5	− 40.0

Lower Chord Stresses.

Members.	2–4	4–6	6–8	8–10
Max.	+44.8	+114.6	+152.8	+174.0
Min.	+ 8.8	+ 23.8	+ 33.8	+ 38.8

Diagonal Stresses.

Members.	1–2	1–4	3–6	5–8	7–10
Max.	−100.1	+100.1	+75.5	+52.4	+35.2
Min.	− 19.5	+ 19.5	+ 8.0	− 5.0	−19.4

Prob. 109. A deck Pratt truss, Fig. 50, has 10 panels, each 12 feet long and 12 feet deep; the dead and train loads are 1000 lbs. and 2000 lbs. per foot per truss, and there are two excess loads 48 feet apart, each of 30 tons: find the stresses in all the members.

Chord Stresses.

Members.	1–3	3–5	5–7	7–9	9–11
Max. . . .	−123.0	−216.0	−279.0	−312.0	−315.0
Min.	− 27.0	− 48.0	− 63.0	− 72.0	− 75.0

Stresses in Diagonals.

Members.	1–4	3–6	5–8	7–10	9–12	8–9	10–11
Max. . .	+173.9	+141.7	+111.1	+82.3	+55.1	+7.2	+29.7
Min. . .	+ 38.0	+ 23.8	+ 7.6	0.0	0.0	0.0	0.0

STRESSES IN VERTICALS.

MEMBERS.	1–2	3–4	5–6	7–8	9–10	11–12
Max.	−138.0	−123.0	−100.2	−78.6	−58.2	−48.0
Min.	− 30.0	− 27.0	− 16.8	− 6.0	− 6.0	− 6.0

Prob. **110.** A deck Whipple truss, Fig. 37, has 12 panels, each 12 feet long and 24 feet deep; the dead and train loads are 1000 lbs. and 3000 lbs. per foot per truss, and there are two excess loads 48 feet apart, each of 30 tons: find the stresses in all the members.

Dead panel load = 6 tons.
Train panel load = 18 tons.
Each excess load = 30 tons.

$$\tan\theta = 0.5;\ \tan\theta' = 1;\ \sec\theta = 1.117;\ \sec\theta' = 1.414.$$

Max. stress in 4–6 $=[3\times24+\frac{30}{12}(11+7)]\times.5=58.5$ tons.

Max. stress in 6–8 $=4\times24+2.5(10+6)1\frac{1}{2}=156.0$ tons.

Max. stress in 3–8

$$=[2\times6+1.5\times25+2.5\times14]1.414=119.5 \text{ tons.}$$

Min. stress in 5–10 $=[9-1.5\times2-2.5\times2]1.414=1.4$ tons.

CHORD STRESSES.

MEMBERS.	2–4	4–6	6–8	8–10	10–12	12–14	9–11
Max. .	+00.0	+58.5	+156.0	+211.5	+285.0	+316.5	−326.0
Min. .	0.0	+ 9.0	+ 24.0	+ 36.0	+ 45.0	+ 51.0	− 54.0

STRESSES IN DIAGONALS.

MEMBERS.	1–4	1–6	3–8	5–10	7–12	9–14	11–12′	13–10	11–8
Max. . .	+130.7	+141.4	+119.5	+97.6	+77.8	+58.0	+40.3	+22.6	+10.6
Min.	+ 20.1	+ 21.2	+ 11.3	+ 1.4	0.0	0.0	0.0	0.0	0.0

STRESSES IN VERTICALS.

MEMBERS.	1–2	3–4	5–6	7–8	9–10	11–12	13–14
Max. . .	–179.0	–117.0	–100.0	–84.5	–69.0	–55.0	–54.0
Min. . .	– 36.0	– 18.0	– 15.0	– 8.0	– 6.0	– 6.0	– 6.0

Prob. 111. A through parabolic bowstring truss, Fig. 44, has 8 panels, each 10 feet long, and 10 feet center depth; the verticals are ties, and the diagonals are struts; the dead and train loads are 1000 lbs. and 3000 lbs. per foot per truss, and there are two excess loads 50 feet apart, each of 30 tons: find the stresses in all the members.

Dead panel load $=$ 5 tons.
Train panel load $= 15$ tons.
Each excess load $= 30$ tons.

Length of 3–4 $= 4.375$ feet; length of 5–6 $= 7.5$ feet.

Length of 7–8 $= 9.375$ feet; length of 9–10 $= 10.0$ feet.

Then, for the maximum chord stresses, a full dead and train panel load must be at each lower apex. For any member of the lower chord, as 2–4, one excess load of 30 tons is placed at 4 and the other at 6′, or 50 feet to the right of the first, and the dead and train loads cover the whole span, as just stated. Here we cannot use the "method of chord increments," since the chords are not parallel, but must use the "method of moments," (1) of Art. 19.

Reaction at the left end $= 3\frac{1}{2} \times 20 + 30\,(\frac{7}{8} + \frac{2}{8}) = 103.75$ tons.

$$\text{Max. stress in } 2\text{–}4 = \frac{103.75 \times 10}{4.375} = 237.1 \text{ tons.}$$

This tensile stress in 2–4 is equal to the horizontal component of the compressive stress in 2–3, by (1) of Art. 5. Therefore the stress in 2–3 is equal to the stress in 2–4 multiplied by the secant of the angle between 2–3 and 2–4. Thus,

$$\text{Max. stress in } 2\text{–}3 = -\,237.1 \times 1.092 = -\,258.9 \text{ tons.}$$

For the maximum stress in 3–5, one excess load is put at 6 and the other at 4′, while the dead and train loads cover the whole span; then take center of moments at intersection of 4–5 and 4–6 or at 4. And so on for the stresses in 5–7 and 7–9.

The diagonal stresses are found by putting only the train and excess loads on the truss in the proper position for each member, since the dead load produces no stress in the diagonals (Art. 34). Thus, to find the maximum stress in 4–5, one excess load is placed at 6 and the other 50 feet to the right of it at 4′, and the train loads cover the span from 6 to the right abutment.

Now cut 3–5, 4–5, and 4–6, and take center of moments at the intersection of 3–5 and 4–6, which is 4 feet to the left of 2; the diagonal 3–6 is not in action for this loading.

$$\text{Lever arm of } 4\text{–}5 = 14 \sin\theta = 14 \times \frac{7.5}{12.5} = 8.4 \text{ feet.}$$

Left reaction for this loading

$$= \frac{15}{8}(1 + 2 + \cdots + 6) + \frac{30}{8}(6 + 1) = 65.625 \text{ tons.}$$

$$\therefore \text{max. stress in } 4\text{–}5 = -\,\frac{65.625 \times 4}{8.4} = -\,31.25 \text{ tons.}$$

UPPER CHORD STRESSES.

MEMBERS.	2–3	3–5	5–7	7–9
Max.	−258.8	−230.5	−213.7	−208.3
Min.	− 43.6	− 41.9	− 40.7	− 40.1

LOWER CHORD STRESSES.

MEMBERS.	2–4	4–6	6–8	8–10
Max.	+237.1	+230.0	+220.0	+220.0
Min.	+ 40.0	+ 40.0	+ 40.0	+ 40.0

STRESSES IN THE DIAGONALS.

MEMBERS.	4–5	6–7	8–9	3–6	5–8	7–10
Max.	−31.3	−34.2	−38.2	−49.0	−43.75	−41.1
Min.	0.0	0.0	0.0	0.0	0.0	0.0

Max. stresses in the verticals $= +50.0$ tons.
Min. stresses in the verticals $= +5$ tons.

Prob. **112.** A through circular bowstring truss, Fig. 45, has 6 panels, each 15 feet long, its web members all forming isosceles triangles, with the upper apexes on the circumference of a circle whose radius is 75 feet, the center depth of the truss being $14\frac{5}{8}$ feet; the dead and train loads are 720 lbs. and 2176 lbs. per foot per truss, and there are two excess loads 45 feet apart, each of 24 tons: find the stresses in all the members.

Dead panel load $= 5.4$ tons.
Train panel load $= 16.32$ tons.
Each excess load $= 24$ tons.

CHORD STRESSES.

MEMBERS.	2–3	3–5	5–7	7–7′	2–4	4–6	6–8
Max. . . .	–149.6	–163.0	–149.2	–137.1	+125.0	+130.8	+125.4
Min. . . .	– 24.7	– 27.2	– 25.6	– 25.0	+ 20.6	+ 22.9	+ 23.7

WEB STRESSES.

MEMBERS.	3–4	5–6	7–8	7′–6′	5′–4′
Max.	+29.9	+33.4	+34.8	+37.2	+43.2
Min.	+ 4.9	–11.9	–15.1	–13.9	– 7.1

Prob. **113.** A through Pratt truss, Fig. 33, has 10 panels, each 12 feet long and 12 feet deep; the dead and train loads are 1000 lbs. and 3000 lbs. per foot per truss, and there are two excess loads 48 feet apart, each of 30 tons: find the stresses in all the members.

Art. 43. The Baltimore Truss.—This truss, Fig. 54, or some modification of it, is now used very generally for long spans when it is desired to avoid long panel lengths. Each large panel is divided into two smaller ones by insert-

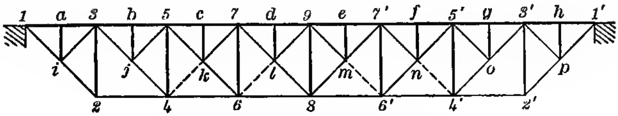


Fig. 54

ing half ties and subvertical struts, or half struts and subvertical ties, according as it is a deck or a through truss. In the deck form, Fig. 54, the subverticals *ai*, *bj*, *ck*, etc., are strained only by the panel loads which they directly

support; the dotted diagonals $4k$, $6l$, $6'm$, $4'n$ are counters, not being in action for full load.

Prob. **114.** A deck Baltimore truss, Fig. 54, has 16 panels, each 10 feet long and 20 feet deep; all the verticals are posts, and all the inclined pieces are ties; the dead and train loads are 800 lbs. and 1600 lbs. per foot per truss, and there are two excess loads 50 feet apart, each of 30 tons: find the stresses in all the members.

$$\text{Dead panel load} = 4 \text{ tons.}$$
$$\text{Train panel load} = 8 \text{ tons.}$$
$$\text{Each excess load} = 30 \text{ tons.}$$

The maximum chord stresses occur when the dead and train loads act at every upper apex (Art. 20), and the excess loads act at the proper apexes (Art. 42). Thus for $3\,b$, we have 30 tons at b and at 9, 50 feet to the right of b. The center of moments is at 4, the intersection of 3–4 and 2–4. Hence, the left reaction for dead and live loads is,

$$R = 7\tfrac{1}{2} \times 12 + \tfrac{30}{16}(13 + 8) = 129.375 \text{ tons.}$$

Calling S the stress in $3\,b$, we have

$$129.375 \times 40 - 12\,(30 + 20) + S \times 20 = 0;$$

$$\therefore S = \text{stress in } 3\,b = -\,228.8 \text{ tons} = \text{stress in } b\,5.$$

The stresses in $1\,a$ and $a\,3$, $3\,b$ and $b\,5$, $5\,c$ and $c\,7$, $7\,d$ and $d\,9$, etc., must always be the same, since the posts ai, bj, ck, etc., are perpendicular to the chords, and cannot therefore cause any stresses in them.

For $5\,c$ and $c\,7$, we must put the excess loads at c and $7'$ and take the center of moments at 6, the intersection of $5\,k$ and 4–6; and so on.

For any lower chord member, as 4–6, we have 30 tons at 5 and at e, 50 feet to the right of 5. The center of moments is at 5. Thus,

$$R = 7\tfrac{1}{2} \times 12 + \tfrac{30}{16}(12 + 7) = 125.625 \text{ tons.}$$

$$\therefore 125.625 \times 40 - 12(30 + 20 + 10) - S \times 20 = 0.$$

$$\therefore S = \text{stress in } 4\text{–}6 = 215.3 \text{ tons.}$$

It is evident that the maximum stress in each subvertical, ai, bj, ck, dl, etc., is equal to a full panel load, $4 + 8 + 30 = 42$ tons compression.

It is also evident that half of the load on ai, bj, ck, etc., is transmitted to 3, 5, 7, etc., through the half ties $i\,3$, $j\,5$, $k\,7$, etc.

Therefore the maximum tension in each of the half ties $i\,3$, $j\,5$, $k\,7$, etc., $= \dfrac{4 + 8 + 30}{2} \sec\theta = 21 \times 1.414 = 29.7$ tons.[1]

The maximum stress in the upper part of any main tie, as $5\,k$, occurs when the excess loads act at c and at $7'$, 50 feet to the right of c, and the train loads are at all the apexes on the right of $5\,k$. The shear in $5\,k$ for this loading is,

$$3\tfrac{1}{2} \times 4 + \tfrac{30}{16}(11 + 6) + \tfrac{8}{16}(11 + 10 + \cdots + 1) = 78.875.$$

$$\therefore \text{max. stress in } 5\,k = 78.875 \times 1.414 = 111.5 \text{ tons.}$$

The maximum stress in any main vertical, as 5–4, is equal to the greatest shear in 5–4; and this greatest shear

[1] At the *center* of the truss these ties must act as *counters*. Thus, take the tie l–9, put the excess loads at 3 and d, and the train loads at all the apexes from the left abutment to d inclusive. Then the greatest shear in the section cutting d–9, l–9, and l–8 is -28.875 tons. As l–8 is not in action for this loading, this is the vertical component of the stress in l–9.

$$\therefore \text{max. stress in } l\text{–}9 = 28.875 \times 1.414 = +40.8 \text{ tons.}$$

in 5–4 occurs when the excess loads act at 5 and at e, 50 feet to the right of 5, and the train loads are at all the apexes from the right abutment to 5 inclusive. Then the greatest shear in 5–4 is that which is due to this loading, together with the four dead loads at 5, c, 7, and d, half of the dead load at 9, and half of the dead load at b, which is transmitted to 5 through the half tie $j\,5$. Hence,

$$\text{Max. stress in } 5\text{–}4 =$$
$$-\left[5 \times 4 + \tfrac{30}{16}(12+7) + \tfrac{8}{16}(12 + \cdots + 1)\right]$$
$$= -\,94.6 \text{ tons.}$$

The maximum stress in the lower part of any main tie, as $j\,4$, is equal to the maximum stress in 4–5 multiplied by the secant of the angle which the tie makes with the vertical. Hence,

$$\text{Max. stress in } j\,4 = 94.6 \times 1.414 = 133.8 \text{ tons.}$$

The maximum stress in 6–l occurs when the excess loads are at a and 7, and the train loads are at all the joints from the left abutment to 7 inclusive. Then the shear in the section cutting d–9, l–9, and l–8 is -21.625 tons, which is the vertical component of the stress in l–9, since l–8 is not in action. The load at d produces in l–9 a negative shear of 2 tons, so that the difference, or 19.625, must come from the member 6–l.

$$\therefore \text{ max. stress in } 6\text{–}l = +\,19.625 \times 1.414 = +\,27.7 \text{ tons.}$$

Otherwise as follows: The live loads going to the right abutment through the panel 6–8 = 23.63 tons. The dead loads crossing this at the section cutting d–9, l–9, and l–8 are $\frac{1}{2}$ of load at 9 + $\frac{1}{2}$ of load at d = 4 tons. The difference is 19.63 tons, for which the panel 6–8 must be counterbraced.

Thus are found the following maximum stresses:

MAXIMUM CHORD STRESSES.

1–3	3–5	5–7	7–9	2–4	4–6	6–8
−136.9	−228.8	−281.6	−295.6	+127.1	+215.3	+264.4

MAXIMUM STRESSES IN UPPER PART OF MAIN TIES AND HALF TIES.

1–*i*	3–*j*	5–*k*	7–*l*	3–*i*	5–*j*	7–*k*	9–*l*
+193.5	+151.1	+111.5	+74.8	+29.7	+29.7	+29.7	+40.8

MAXIMUM STRESSES IN LOWER PART OF MAIN TIES.

i 2	*j*–4	*k*–6	*l*–8	6–*l*	4–*k*
+174.8	+133.8	+95.6	+60.3	+27.7	+0.7

MAXIMUM STRESSES IN THE VERTICALS.

2–3	4–5	6–7	8–9	*ai*, *bj*, *ck*, *dl*.
−123.6	−94.6	−67.6	−59.6	−42.0

NOTE.—In the above determination of the maximum stresses in the long verticals and in the lower ends of the long diagonals, as, for example, in 5–4 and *j*–4, it was assumed that the largest possible shear in 5–4 or in *j*–4 occurs when the excess loads act at 5 and at *e*, and the train loads are at all the apexes from the right abutment to the joint 5 inclusive. It is evident, however, from a little inspection of Fig. 54, that, if a train panel load be placed at the joint *b*, one half of this train load at *b* will be transmitted to the joint 5 through the half tie *j* 5, just as one half of the dead load at *b* is transmitted to 5 through the half tie *j* 5, and that this half train load transmitted to 5, minus

the part that is transmitted to the right abutment, which is $\frac{3}{16}$ of the train load at b, will form part of the shear in 5–4 or in j–4; that is, the shear will be increased by $\frac{1}{2} \times 8 - \frac{3}{16} \times 8 = \frac{5}{16} \times 8 = 2.5$. Hence,

$$\text{Max. stress in } 5\text{–}4 = -[5 \times 4 + \tfrac{10}{16}(12 + 7) + \tfrac{8}{16}(12 + \cdots + 1) + 2.5]$$
$$= -\ 97.1 \text{ tons.}$$

Max. stress in j–4 $= 97.1 \times 1.414 = 137.3$ tons.

Similarly,

Max. stress in 2–3 $= -[7 \times 12 + \frac{10}{16}(14 + 9)] = -\ 127.1$ tons.

Max. stress in i–2 $= 127.1 \times 1.414 = 179.7$ tons,

and so on for 6–7 and k–6, 8–9, and l–8.

Thus, stress in 6–7 $= -\ 68.6$, in k–6 $= 97.0$, in 8–9 $= -\ 43.1$, in l–8 $= 61$ tons.

It is evident that by putting a train panel load on the apex just to the left of the section considered, the shear in that section is increased the most when near the left end of the truss; and that it decreases on approaching the middle of the truss without becoming zero.

Prob. 115. A deck Baltimore truss, Fig. 55, has 14 panels, each 10 feet long and 20 feet deep, the verticals being posts and the diagonals ties; the dead and train loads are 1000 lbs.

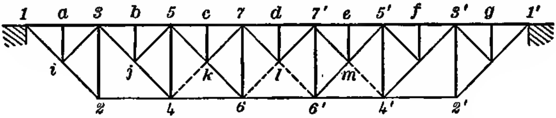


Fig. 55

and 2000 lbs. per foot per truss, and there are two excess loads 50 feet apart, each of 30 tons: find the maximum stresses in all the members. (See note to last problem.)

MAXIMUM CHORD STRESSES.

1–3	3–5	5–7	7–7′	2–4	4–6	6–6′
−142.5	−230.4	−271.1	−264.7	+130.7	+214.3	+250.7

MAXIMUM STRESSES IN UPPER PART OF MAIN TIES AND HALF TIES.

1–*i*	3–*j*	5–*k*	7–*l*	3–*i*	5–*j*	7–*k*
+201.5	+150.0	+102.5	+59.1	+31.8	+31.8	+31.8

MAXIMUM STRESSES IN LOWER PART OF MAIN TIES.

i–2	*j*–4	*k*–6	6–*l*	4–*k*
+184.8	+133.3	+85.8	+42.4	+8.1

MAXIMUM STRESSES IN THE VERTICALS.

2–3	4–5	6–7	*ai*, *bj*, *ck*, *dl*.
−130.7	−94.3	−60.7	−45.0

Prob. 116. A through Baltimore truss, Fig. 56, has 16 panels, each 16 feet long and 32 feet deep; the dead load is given by formula (3), Art. 15, the train load is 1500 lbs.

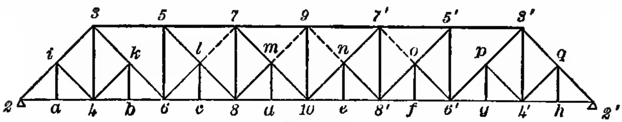


Fig. 56

per foot per truss, and there are two excess loads 48 feet apart, each of 30 tons: find the maximum stresses in all the members.

$$\text{Dead panel load} = \left(\frac{5 \times 256 + 750}{2 \times 2000}\right)16 = 8 \text{ tons.}$$

Train panel load = 12 tons.

Each excess load = 30 tons.

The maximum chord stresses occur when the dead and train loads act at every lower apex (Art. 20), and the excess loads act at the proper apexes (Art. 42). Thus for $b6$, we have 30 tons at 4 and at c, 48 feet to the right of 6. The center of moments is at 3, the intersection of 3–5 and 3–6. Hence the left reaction for dead and live loads is,

$$R = 7\tfrac{1}{2} \times 20 + \tfrac{30}{16}(14 + 11) = 196.875 \text{ tons.}$$

Calling s the max. stress in $b6$, we have,

$$196.875 \times 32 - 20 \times 16 + 20 \times 16 - s \times 32 = 0.$$

$\therefore\ s =$ max. stress in $b6 = 196.9$ tons $=$ max. stress in $4b$.

For any upper chord member, as 5–7, we have 30 tons at 8 and at e, 48 feet to the right of 8. The center of moments is at 8. Thus,

$$R = 7\tfrac{1}{2} \times 20 + \tfrac{30}{16}(10 + 7) = 181.875 \text{ tons.}$$

$$\therefore\ 181.875 \times 6 \times 16 - 20(16 + 32 + 48 + 64 + 80) + s \times 32 = 0.$$

$$\therefore\ s = \text{max. stress in 5–7} = -\ 395.6 \text{ tons.}$$

Similarly, the other chord stresses are found.

It is evident that the maximum stress in each subvertical, ai, bk, cl, dm, etc., is equal to a full panel load $8 + 12 + 30 = 50$ tons tension.

It is also evident that half of the load on ai, bk, cl, dm, etc., is transmitted to 4, 4, 6, 8, etc., through the half struts $i4$, $k4$, $l6$, $m8$, etc.

Therefore the maximum compression in each of the half struts $i4$, $k4$, $l6$, $m8$, etc.

$$= \frac{8 + 12 + 30}{2} \sec\theta = 25 \times 1.414 = 35.4 \text{ tons.}$$

The maximum stress in the lower part of any main tie, as $l8$, occurs when the excess loads act at 8 and at e, 48 feet to the right of 8, and the train loads are at all the apexes

from the right abutment to 8 inclusive. The shear in $l8$ for this loading is,

$$2\tfrac{1}{2} \times 8 + \tfrac{30}{16}(10 + 7) + \tfrac{12}{16}(10 + \cdots + 1) = 93.125.$$

$$\therefore \text{ max. stress in } l8 = 93.125 \times 1.414 = 131.7 \text{ tons.}$$

The maximum stress in the upper part of any main tie, as $3k$, occurs when the excess loads act at 6 and at d, 48 feet to the right of 6, and the train loads are at all the apexes from the right abutment to b inclusive: for half of the train load at b is transmitted to the joint 6 through the half strut $k6$, and $\frac{3}{16}$ of the train load at b is transmitted to the right abutment; therefore the difference of these, or $\frac{5}{16}$ of the train load at b, goes to the left abutment and forms part of the shear in $3k$; that is, the train panel load at b will increase the shear in $3k$ by $\frac{5}{16} \times 12 = 3\frac{3}{4}$ tons. It is evident also that half of the dead load at b, which is transmitted to 6 through the half strut $k6$, goes to the left abutment and forms part of the shear in $3k$. Similarly for $5l$ and $7m$. Hence,

Max. stress in $3k$

$$= [5 \times 8 + \tfrac{30}{16}(12 + 9) + \tfrac{12}{16}(12 + \cdots + 1) + 3.75]\,1.414$$
$$= 200.3 \text{ tons.}$$

The maximum stress in 3–4 is equal to the full panel load at 4 plus half the sum of the panel loads at a and b

$$= 30 + 12 + 8 + \tfrac{1}{2}(24 + 16) = 70 \text{ tons.}$$

The maximum stress in either of the other long verticals, as 7–8, is equal to the shear in $7m$ plus whatever load is applied to the upper apex 7. Hence,

Max. stress 7–8

$$= -[8 + \tfrac{30}{16}(8 + 5) + \tfrac{12}{16}(8 + \cdots + 1) + \tfrac{1}{16} \times 12]$$
$$= -60.1 \text{ tons.}$$

The maximum stress in $3i$ occurs when the excess loads act at 4 and at c, and the train loads cover the whole truss; it is easily seen that half of the dead train loads at a, which are transmitted to 4, form part of the shear in $3i$. Hence,

$$\text{Max. stress in } 3i = -[7 \times 20 + \tfrac{30}{16}(14 + 11)]\,1.414$$
$$= -264.2 \text{ tons.}$$

The following stresses are found in a manner similar to the above:

MAXIMUM CHORD STRESSES.

3–5	5–7	7–9	2–4	4–6	6–8	8–10
−318.8	−395.6	−417.5	+196.9	−196.9	+328.8	+405.6

MAXIMUM STRESSES IN LOWER PART OF MAIN AND HALF DIAGONALS.

i–2	k–6	l–8	m–10	i–4	k–4	l–6	m–8
−283.7	+189.3	+131.7	+78.3	−35.4	−35.4	−35.4	−35.4

MAXIMUM STRESSES IN UPPER PART OF MAIN DIAGONALS.

3–i	3–k	5–l	7–m	9–m	7–l
−264.2	+200.3	+140.5	+85.0	+53.2	+6.2

MAXIMUM STRESSES IN THE VERTICALS.

3–4	5–6	7–8	9–10	ai, bk, cl, dm.
+70.0	−99.4	−60.1	−23.9	+50.0

The stress in $9-m = 9-n$ occurs when the excess loads act at e and $6'$, and the train loads act at all the joints from the right abutment to e. Then the shear in the section cutting $9-7'$, $9-n$, and $10-n = 101.625 - 8 \times 8 = +37.625$ tons; and as $10-n$ cannot take compression, this is the vertical component of stress in $9-n$.

$\therefore$ max. stress in $9-n = +37.625 \times 1.414 = +53.2$ tons.

If $10-n$ were constructed to take compression, this would be greatly modified. In this case the excess loads would act at $8'$ and g, and the train loads at all the joints from the right abutment to $8'$. Then the left reaction $= 92.625$ tons and the shear in the section cutting $9-7'$, $9-n$, and $10-n = 92.625 - 64 = +28.625$ tons. The member $10-n$ takes a positive shear of 4 tons, so that the vertical component of stress in $9-n = +28.625 - 4 = +24.625$ tons.

$\therefore$ max. stress in $9-n = +24.625 \times 1.414 = +34.8$ tons.

The stress in $7'-0$ occurs when the excess loads act at f and $4'$, and the train loads at all the joints from the right abutment to f. Then the left reaction $= +84.375$ tons and the shear in the section cutting $7'-5'$, $7'-0$, and $8'-0 = 84.375 - 8 \times 10 = +4.375$ tons. As $8'-0$ does not act in compression, this shear must be resisted by $7'-0$ alone.

$\therefore$ max. stress in $7'-0 = +4.375 \times 1.414 = +6.2$ tons.*

Prob. **117.** A deck Baltimore truss, Fig. 54, has 16 panels, each 10 feet long and 20 feet deep, the verticals being posts and the diagonals ties; the dead and train loads are 1000 lbs. and 2000 lbs. per foot per truss, and there are two excess loads 50 feet apart, each of 33 tons: find the stresses in all the members.

*It should be noted that the members $m-8$ and $n-8'$ are also tension members for counter stresses.

The tension stress in each of these members $= 20.625 \times 1.414 = 29.2$ tons.

MAXIMUM CHORD STRESSES.

1–3	3–5	5–7	7–9	2–4	4–6	6–8
−164.1	−274.1	−337.7	−354.7	+152.4	+258.4	+317.8

MAXIMUM STRESSES IN UPPER PART OF MAIN AND HALF TIES.

1–*i*	3–*j*	5–*k*	7–*l*	3–*i*	5–*j*	7–*k*	9–*l*
+232.0	+180.6	+132.7	+88.3	+34.0	+34.0	+34.0	+47.4

MAXIMUM STRESSES IN LOWER PART OF MAIN TIES.

i–2	*j*–4	*k*–6	*l*–8	6–*l*
+215.5	+164.1	+116.2	+69.2	+31.8

The counter 4–*k* is found not to be required; but in actual practice a counter would be used to provide for loads differing from the ones above assumed.

MAXIMUM STRESSES IN THE VERTICALS.

2–3	4–5	6–7	8–9	*ai*, *bj*, *ck*, *dl*.
−152.4	−116.1	−82.2	−69.2	+48.0

This problem is taken from "Strains in Framed Structures" by A. J. Du Bois. It will be seen that the answers above given for the maximum stresses in the lower part of the main ties and in the long verticals differ somewhat from those given by Prof. Du Bois.

Art. 44. True Maximum Shears for Uniform Live Load. — Thus far, it has been assumed that the live panel loads were all concentrated at the panel points, and in order to find the maximum stress in any diagonal due to a uniform live load per linear foot, the maximum live load shear in that diagonal has been found by putting a live panel load on every joint on the right of the section cutting the diagonal (Art. 22).

For example, it has been assumed that the first apex on the right of the section in Fig. 57 has a full live panel load and the first one on the left has no live load. Now the joint n on the right of the section receives the half panel load

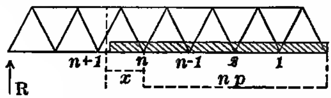


Fig. 57

between the joints n and $n-1$, but only a small part of the load on the left of n; the other part of the load on the left of n is received by the joint $n+1$. When the uniform load extends from the right abutment to the joint $n+1$, the joint n receives a full panel load; but then the joint $n+1$ receives a half panel load. Let us therefore ascertain at what distance x to the left of the joint n the load must extend to produce the greatest shear in the $n+1th$ panel.

Let $N =$ the number of panels in the truss, $p =$ the panel length, $w =$ the uniform live load per linear foot, $n =$ the number of whole panels covered by the load, $R =$ the reaction at the left or unloaded end. Then,

$\frac{wx^2}{2p} =$ the part of the load wx which is carried by the joint $n+1$; and $wx - \frac{wx^2}{2p} =$ the part of the load wx which is carried by the joint n.

We have then for the reaction

$$R = \frac{wx^2}{2p}\frac{(n+1)}{N} + \left(wx - \frac{wx^2}{2p}\right)\frac{n}{N} + \frac{wn^2p}{2N}$$

$$= \frac{wxn}{N} + \frac{wx^2}{2pN} + \frac{wn^2p}{2N}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Hence the shear in the $n+1th$ panel is

$$V = \frac{wxn}{N} + \frac{wx^2}{2pN} + \frac{wn^2p}{2N} - \frac{wx^2}{2p}. \quad . \quad . \quad . \quad . \quad (2)$$

Equating the derivative of V to zero, we have,

$$\frac{wn}{N} + \frac{wx}{pN} - \frac{wx}{p} = 0;$$

$$\therefore\ x = \frac{n}{N-1}p,$$

which is the distance to the left of the joint n that the live load must extend in order to produce the maximum shear in the $n+1th$ panel from the right end.

Thus, let the truss have 8 panels; then for the true maximum shear in the sixth panel from the right end $x = \frac{5}{7}p$, for the seventh panel $x = \frac{6}{7}p$, and for the eighth or last panel $x = \frac{7}{7}p = p$, or the whole truss is covered.

The total live load on the truss

$$= nwp + \frac{nwp}{N-1} = \frac{Nnwp}{N-1}$$

$= N$ times the load on the $n+1th$ panel.

Therefore, the live load on the $n+1th$ panel is $\frac{1}{N}th$ of the total live load on the truss.

Substituting the above value of x in (2) and reducing, we have

$$\text{Maximum shear} = \frac{wpn^2}{2(N-1)}.$$

These values of x and the maximum shear are independent of the form of the truss, whether the webbing be inclined, as in the Warren, or inclined and vertical, as in the Pratt and Howe.

Prob. **118.** A truss, Fig. 57, has 7 panels, each 16 feet long, the live load is 2 tons per foot: find (1) the true maximum live load shears in every panel and (2) the maximum live load shears by the usual method (Art. 22).

Here $N = 7$, $p = 16$ feet, and $w = 2$ tons.

For the maximum shear in the 1st panel from the left $n = 6$.

$$\therefore\ x = p = 16 \text{ feet, and shear} = \frac{2 \times 16 \times 6^2}{2 \times 6} = 96 \text{ tons,}$$

which is half of the effective load, or the effective reaction.

For max. shear in the 4th panel from left, $n = 3$.

$\therefore\ x = \frac{3}{6} \times 16 = 8$ feet, or the load reaches to the middle of the 4th panel.

$$\text{Shear} = \frac{2 \times 16 \times 3^2}{2 \times 6} = 24.0 \text{ tons.}$$

By the usual way of taking a full panel load as concentrated at the third apex from the right $n - 1$, and none at the 4th apex n, we have

$$\text{Shear} = \tfrac{32}{7}(1 + 2 + 3) = 27.4 \text{ tons,}$$

which is greater than that obtained by the strictly correct method.

Thus the following live load shears are computed (1) *by the Strictly Correct Method* and (2) *by the Method in Common Practice:*

True Method. 96.0, 66.7, 42.7, 24.0, 10.7, 2.7, 0.0 tons.

Common Method. 96.0, 68.6, 45.7, 27.4, 13.7, 4.6, 0.0 tons.

It is seen that the shears obtained by the *usual method*, that is, by supposing the panel loads to be concentrated at the panel points, are larger than those obtained by the *true method*. For the reason that the shears obtained by the usual method always err on the safe side, and that the error is always small, it is the common practice to suppose all the load from middle to middle of panel to be concentrated at the panel point, as in Art. 4.

Art. 45. Locomotive Wheel Loads. — In computing the stresses in railway bridges in America, the live load which is now generally taken, consists of two of the heaviest engines in use on the line, at the head of the heaviest known train load. The weights of the engines and tenders are assumed to be concentrated at the wheel-bearings, giving definite loads at these points, while the train load is taken as a uniform load of about 3000 pounds per linear foot of single track.

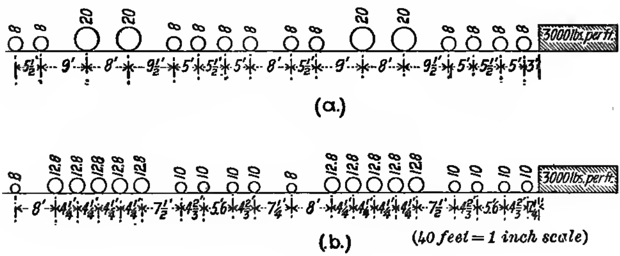


Fig. 58.

The first diagram of Fig. 58 represents two 88-ton passenger locomotives, as specified by the Pennsylvania Railroad.

The second diagram shows two 112-ton decapod engines, used on the Atlantic Coast Line.

The numbers above the wheels show their weights in tons for both rails of a single track. The numbers between the wheels show their distances apart in feet.

Art. 46. Position of Wheel Loads for Maximum Shear.—Let Fig. 59 represent a truss with a decapod engine and train upon it.

Let N = the number of panels in the truss, p = the panel length, $P = P_1 + P_2 + \cdots + P_{10}$ = the sum of the wheel loads, w = the uniform train load per linear foot, W = the total live load on the truss, d = the distance of the center of

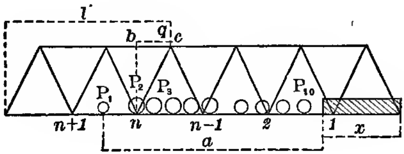


Fig. 59

gravity of P from the front of the train, x = the distance of the front of the train from the right abutment, a = the distance of P_1 from the front of the train, y = its distance at the left of the nth panel point, and R = the reaction at the left end. Then $P_1 \frac{y}{p}$ = the part of the load P_1 that is carried by the $n + 1th$ panel point.

We have then for the reaction

$$R = \frac{P(x+d)}{Np} + \frac{wx^2}{2\,Np}. \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (1)$$

Hence the shear in the $n + 1th$ panel is

$$V = \frac{P(x+d)}{Np} + \frac{wx^2}{2\,Np} - P_1 \frac{y}{p}. \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (2)$$

But from the figure, $x + a = np + y$; $\therefore\ y = a + x - np$, which in (2) gives

$$V = \frac{P(x+d)}{Np} + \frac{wx^2}{2\,Np} - \frac{P_1}{p}(a + x - np).$$

Equating the first derivative of V to zero, we have,

$$\frac{P + wx}{N} - P_1 = 0;$$

or, since $P + wx = W$, we have,

$$P_1 = \frac{1}{N} W.$$

Hence, the shear in any panel is a maximum when the load on the panel is $\frac{1}{N}$*th of the entire live load on the truss.*

In practice it is convenient to put one of the loads at the nth panel point, so that the above condition cannot, in general, be exactly satisfied. We must have in general,

$$P_1 = \text{or} < \frac{1}{N} W.$$

Hence, in general, *the shear in the* $n + 1$*th panel is a maximum when one of the loads is at the* n*th panel point, and the load on the* $n + 1$*th panel is equal to or just less than* $\frac{1}{N}$*th of the entire load on the truss.*[1]

Prob. **119.** A through Warren truss has 10 panels, each 12 feet long: find the maximum shears in each panel caused by a single decapod engine and tender.

Here each driver in Fig. 59 weighs 12.8 tons, each tender wheel weighs 10 tons, and the pilot wheel weighs 8 tons; the total load W is 112 tons.

[1] This holds good in all cases for shear.

Let it be required to find the maximum shear in the 2d panel from the left. Here $n = 8$; $N = 10$.

By the above rule for the maximum shear in this panel, the first driver must be put at the joint n, since $8 < \frac{112}{10}$, and $8 + 12.8 > \frac{112}{10}$.

In this position of the loads, the left reaction is

$$R = \frac{8}{120} \times 104 + \frac{12.8}{120}(96.0 + 91.75 + 87.5 + 83.25 + 79.0)$$
$$+ \frac{10}{120}(71.5 + 66.83 + 61.23 + 56.56) = 74.9 \text{ tons.}$$

$$\therefore \text{max. shear} = 74.9 - \frac{8 \times 8}{12} = 69.6 \text{ tons.}$$

Let it be required to find the maximum shear in the 8th panel from the left. Here $n = 2$.

To find the position of the wheels for maximum shear in this panel, try the first driver at the joint n, as before.

For this position the total live load on the truss

$$= 8 + 5 \times 12.8 = 72 \text{ tons,}$$

since the tender wheels are off the truss. By the rule, therefore, this is not the correct position, since $8 > \frac{1}{10} \times 72$.

If we put the pilot wheel at n, the load on the truss

$$= 8 + 4 \times 12.8 = 59.2 \text{ tons,}$$

since the last driver is off the truss. Hence for maximum shear in this panel this is the correct loading.

With the live load in this position, the left reaction is

$$R = \frac{8}{120} \times 24 + \frac{12.8}{120}(16 + 11.75 + 7.50 + 3.25) = 5.7 \text{ tons,}$$

which is also the maximum shear, since in this case there is no load to be subtracted.

Thus the following shears are determined:

Max. shears = 80.8, 69.6, 58.4, 47.2, 36.0, 24.8, 13.9, 5.7.

Prob. 120. A deck Pratt truss, Fig. 50, has 10 panels, each 20 feet long and 24 feet deep: find the maximum web stresses in each panel caused by a single passenger locomotive and tender, as shown in (*a*) of Fig. 58.

We first find the maximum shear in the left panel. By the rule the second pilot wheel must be put at the joint 4, since $8 < \frac{88}{10}$, and $8 + 8 > \frac{88}{10}$.

With the live load in this position, the left reaction is

$$R = \frac{8}{200}(185.5 + 180 + 153.5 + 148.5 + 143 + 138)$$
$$+ \frac{20}{200}(171 + 163) = 71.34 \text{ tons};$$

and the max. shear $= 71.34 - 8 \times \frac{5.5}{20} = 69.14$.

$$\therefore \text{stress in } 1\text{–}4 = 69.14 \sec\theta = 69.14 \times 1.302 = 90.0 \text{ tons.}$$

For 3–4 we have the same loading as for 1–4; and the maximum compression in 3–4 is equal to the shear just found for 1–4.

$$\therefore \text{stress in } 3\text{–}4 = -69.1 \text{ tons.}$$

The maximum compression for the end vertical 1–2 is found by placing the wheels so as to bring the greatest load to the joint 1, and is found to be 71.9 tons.

Thus we find the following maximum stresses:

Diagonals $= +90.0,\ +78.6,\ +67.1,\ +55.6,\ +44.2,\ +32.7,$
$+21.2,\ +9.8$ tons.

Posts $= -71.9, -69.1, -60.3, -51.5, -42.7, -33.9$ tons.

If the second driver be placed at the middle panel point 11, the stress in 11–12 will be found to be -39.6 tons.

Prob. 121. A deck Pratt truss, Fig. 50, has 10 panels, each 12 feet long and 12 feet deep; the dead load is 1000

lbs. per linear foot of track, the live load is a passenger locomotive and tender, as shown in (a) of Fig. 58: find the maximum web stresses in each panel.

The dead and live load stresses may be computed separately.

STRESSES IN DIAGONALS.

MEMBERS.	1–4	3–6	5–8	7–10	9–12	11–10′	9′–8′
Dead stresses.	+ 38.0	+ 29.6	+21.1	+12.7	+ 4.2	− 4.2	−12.7
Live stresses .	+ 88.3	+ 75.9	+63.4	+51.0	+38.5	+26.1	+13.7
Max. stresses.	+126.3	+105.5	+84.5	+63.7	+42.7	+21.9	+ 1.0

STRESSES IN THE POSTS.

MEMBERS.	1–2	3–4	5–6	7–8	9–10	11–12
Dead stresses .	− 30.0	−27.0	−21.0	−15.0	− 9.0	− 6.0
Live stresses .	− 70.9	−62.5	−53.7	−44.9	−36.1	−27.3
Max. stresses .	−100.9	−89.5	−74.7	−59.9	−45.1	−33.3

If the first driver be placed at the middle panel point 11, the stress in 11–12 will be found to be − 28.7 tons. It will be noticed that the tension of 4.2 tons in the diagonal 9′–12 is equivalent to a compression of 4.2 tons in the diagonal 11–10′; and similarly for the next diagonal 9′–8′.

Prob. 122. A through Pratt truss, Fig. 60, has 8 panels, each 18 feet long and 24 feet deep: find the maximum web

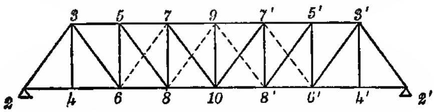


Fig. 60

stresses in each panel due to a passenger locomotive and tender, as shown in (*a*) of Fig. 58, followed by a uniform train load of 3000 lbs. per linear foot.

(1) To find the maximum stress in 2–3.

Try the first driver at the panel point 4. In this position the total live load on the truss

$$= 88 + 90 \times 1.5 = 223 \text{ tons.}$$

Since $8 + 8 < \frac{223}{8}$, and $8 + 8 + 20 > \frac{223}{8}$, this is the correct position for the maximum shear in the panel 2–4. The reaction is

$$R = \frac{8}{144}(135 + 140.5 + 108.5 + 103.5 + 98 + 93)$$

$$+ \frac{20}{144}(126 + 118) + \frac{1}{2} \times \frac{1.5 \times 90^2}{144} = 113.77 \text{ tons.}$$

$$\therefore \text{ max. shear} = 113.77 - \frac{8}{18}(9 + 14.5) = 103.33.$$

$$\therefore \text{ stress in } 2\text{–}3 = -103.33 \times 1.25 = -129.2 \text{ tons.}$$

(2) To find the maximum stress in 9–8′.

Try the first driver at the joint 8′. In this position the total live load on the truss

$$= 88 + 18 \times 1.5 = 115 \text{ tons.}$$

But $8 + 8 > \frac{115}{8}$; hence this is not the correct position for the maximum shear in the panel 10–8′.

We therefore try the second pilot wheel at 8′. We then have, for the total live load on the truss,

$$W = 88 + 9 \times 1.5 = 101.5 \text{ tons.}$$

Since $8 < \dfrac{101.5}{8}$, this is the correct position for the maximum shear in the panel 10–8′.

The reaction is

$$R = \frac{1540 + 1640 + 13.5 \times 4.5}{144} = 22.51 \text{ tons.}$$

$$\therefore \text{ max. shear} = 22.51 - \frac{8 \times 5.5}{18} = 20.1 \text{ tons.}$$

$$= - \text{ stress in } 9\text{–}10.$$

$$\therefore \text{ stress in } 9\text{–}8' = 20.1 \times 1.25 = 25.1 \text{ tons.}$$

The max. tension in the hip vertical 3–4 is found by putting the wheels so as to bring the greatest load at 4. Thus the following maximum stresses are found:

Diagonals

$$= -129.2,\ +96.4,\ +67.9,\ +43.6,\ +25.1,\ +11.3 \text{ tons.}$$

Verticals $= +36.9,\ -54.3,\ -34.9,\ -20.1$ tons.

Art. 47. Position of Wheel Loads for Maximum Moment at Joint in Loaded Chord. — In addition to the notation of Art. 46, let P' = the load on the left of the panel point n, Fig. 59, x' = the distance of its center of gravity from the point n, and n' = the number of panels between the left abutment and the point n. Then for the moment at the panel point n, Fig. 59, we have

$$M = Rn'p - P'x'$$

$$= \left[\frac{P(x+d)}{Np} + \frac{wx^2}{2\,Np}\right] n'p - P'x', \text{ [by (1) of Art. 46].}$$

Equating the first derivative of M to zero, we have, since

$$dx' = dx,\ \frac{(P + wx)\,n'}{N} - P' = 0\,;$$

or since $P + wx = W$ (Art. 46), we have

$$P' = \frac{n'}{N} W.$$

Hence the moment at any panel point in the loaded chord is a maximum when the load on the left of the point is $\frac{n'}{N}$*ths of the entire live load on the truss.*

In practice it is convenient to put one of the loads at the nth panel point, so that the above condition can very seldom be exactly satisfied. We must have in general

$$P' = \text{or} < \frac{n'}{N} W.$$

Hence, in general, *the moment at any panel point of the loaded chord is a maximum when the load on the left of the point has to the entire load on the truss a ratio which is equal to or just less than the ratio which the number of panels on the left of the point bears to the entire number of panels in the truss.*

By this rule the maximum chord stress in any member of the unloaded chord may be determined; since we have only to divide the maximum moment at the panel point opposite the chord member by the depth of truss.

Prob. **123.** A deck Pratt truss, like Fig. 50, has 8 panels, each 18 feet long and 24 feet deep: find the maximum chord stresses due to a single passenger locomotive and tender.

Let it be required to find the maximum stress in the second panel 3–5. Here $n' = 2$, $N = 8$, $W = 88$; therefore, by the rule, P' must be equal to or less than $\frac{2}{8} \times 88$, or 22 tons. Hence, the first driver must be put at 5 since

$$8 + 8 < 22, \text{ and } 8 + 8 + 20 > 22.$$

With the live load in this position, the left reaction is

$$R = \frac{8}{144}(122.5 + 117 + 90.5 + 85.5 + 80 + 75) + \frac{20}{144}(108 + 100) = 60.58 \text{ tons.}$$

$$\therefore \text{stress in } 3\text{–}5 = -\frac{[60.58 \times 36 - 8(9 + 14.5)]}{24} = -83 \text{ tons}$$
$$= - \text{stress in } 6\text{–}8.$$

Thus are found the following stresses:

Max. stress in 1–3 $= -47.7$, in 3–5 $= -83$, in 5–7 $= -103.8$, in 7–9 $= -110.5$ tons.

Prob. 124. A deck Pratt truss, Fig. 50, has 10 panels, each 20 feet long and 24 feet deep: find the maximum chord stresses in each panel due to a passenger locomotive and tender.

Ans. Max. stresses in upper chord $= -57.6$, -103.0, -136.4, -155.2, -161.9 tons.

Prob. 125. A through Warren truss has 10 panels, each 12 feet long and 12 feet deep: find the maximum stresses in the upper chord due to a decapod engine and tender.

Ans. Max. stresses $= -80.8$, -145.0, -190.9, -217.3, -225.2 tons.

Art. 48. Position of Wheel Loads for Maximum Moment at Joint in Unloaded Chord. — By the rule deduced in Art. 47, the maximum chord stresses may be determined in the *unloaded* chord of any simple truss, and also in the *loaded* chord of such trusses as the Pratt and Howe, where the web members are *vertical and inclined* so that the panel points of the upper chord are directly over those of the lower chord. For trusses like the Warren and lattice, where *all* the web members are inclined, it applies only to the *unloaded* chord. For the *loaded chord* of such trusses a modification of the formula or rule is necessary, which may be deduced as follows:

Let it be required to find the maximum moment at the panel point *c* of the unloaded chord, Fig. 59.

Let $W =$ the total live load on the truss, $Q =$ the load on the $(n - 1)$th panel, and $P' =$ the load on the left of the $(n - 1)$th panel.

Let $l' =$ the distance of c from the left support, $q =$ the distance bc, $p =$ the panel length, and $l =$ the length of the span.

Let $x =$ the distance from the center of gravity of W to the right support, $x_1 =$ the distance from the center of gravity of P' to the panel point n, and $x_2 =$ the distance from Q to the panel point $n - 1$.

The part of Q that is carried by the nth panel point is $\dfrac{Qx_2}{p}$.

Hence the moment at c is

$$M = \frac{Wx}{l} l' - P'(x_1 + q) - \frac{Qx_2}{p} q.$$

Equating the first derivative of M to zero, we have, since $dx = dx_1 = dx_2$,

$$\frac{Wl'}{l} - P' - \frac{Qq}{p} = 0. \quad \therefore \ P' + \frac{q}{p} Q = \frac{l'}{l} W. \quad . \quad . \quad . \quad (1)$$

If the braces are equally inclined, that is, if the center of moments c is directly over or under the center of the opposite member, as is usually the case, we have $\dfrac{q}{p} = \dfrac{1}{2}$, and (1) becomes

$$P' + \tfrac{1}{2} Q = \frac{l'}{l} W. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

For the Pratt and Howe trusses $q = 0$, and (1) becomes

$$P' = \frac{l'}{l} W,$$

which is the same as the formula that was found in Art. 47.

It is convenient in practice to put one of the loads at the $n-1$th panel point, so that, *in general*, we must have

$$P' + \tfrac{1}{2}Q = \text{or} < \frac{l'}{l} W. \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

Prob. **126.** A through Warren truss has 10 panels, each 12 feet long and 12 feet deep: find the maximum stresses in the lower chord due to a decapod engine and tender.

To find the maximum stress in the first panel 2–4. Here $l' = \frac{1}{2}$, $l = 10$, and $W = 112$. Therefore by formula (3), $P' + \frac{1}{2}Q$ must be equal to or less than $\frac{1}{20} \times 112$ or 5.6 tons. Hence the 1st driver must be put at 4, since

$$\tfrac{1}{2} \times 8 < 5.6, \text{ and } 8 + \tfrac{1}{2} \times 12.8 > 5.6.$$

The left reaction then = 86.14 tons.

The moment at the point 1 is

$$M = 86.14 \times 6 - 8 \times \tfrac{8}{12} \times 6 = 484.84.$$

$$\therefore \text{ stress in } 2\text{–}4 = \frac{484.84}{12} = 40.4 \text{ tons.}$$

Similarly the maximum stresses in the other lower chord members are found.

Max. stresses = 484.8, 111.9, 167.1, 203.4, 220.7 tons.

Sug. To find stress in 4–6, put 2d driver at 6; in 6–8, put 3d driver at 8; in 8–10, put 4th driver at 10; in 10–12, put 5th driver at 12.

Art. 49. Tabulation of Moments of Wheel Loads.—A diagram such as is shown in Fig. 61, diminishes considerably the work of computing stresses due to actual wheel loads. The first diagram is for an 88-ton passenger locomotive and its tender; the second is for a 112-ton decapod engine and its tender. Any locomotive can be diagramed

in the same manner; and the same method applies to two locomotives coupled together.

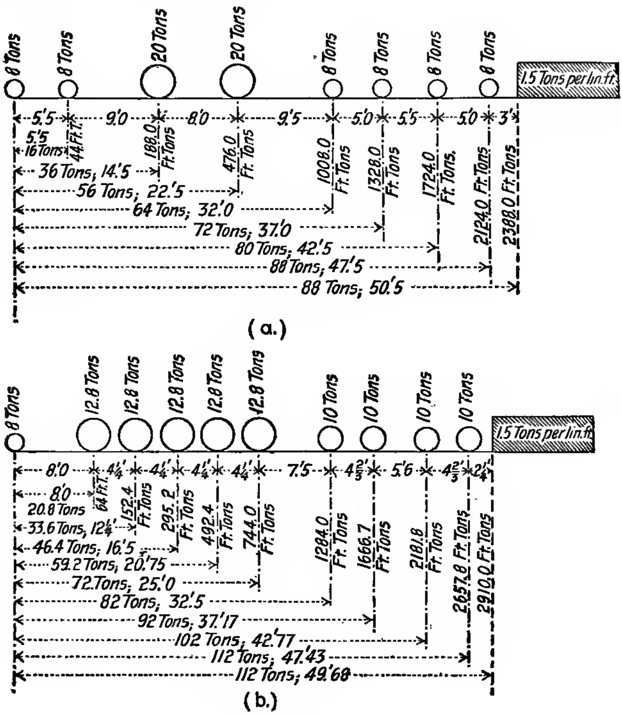


Fig. 61.

(20 feet = 1 inch scale.)

Each diagram shows the weights and distances apart of the wheels. Below each wheel, on the *horizontal* line, is shown the weight of that wheel together with that of all the preceding ones, and its distance from the front wheel.

Below each wheel on the *vertical* line is given the moment of all the preceding wheels, with reference to that wheel. Thus, for the third driver of the decapod engine, we have 46.4 tons for the weight of it and the three preceding wheels, 16.5 feet for its distance from the front wheel, and 295.2 foot-tons for the moment of the preceding wheels with reference to the third driver. At the beginning of the uniform load we have 112 tons for the weight of all the preceding loads, 49.68 feet for the distance of this point from the front wheel, and 2910 foot-tons for the moment of all the preceding loads with reference to this point. Each diagram shows also that, *the moment at any wheel is equal to the moment at the next preceding wheel on the left, plus the sum of all the preceding wheel loads multiplied by the distance from the next left preceding wheel to the wheel in question.* Thus, in (a) the moment of the first three wheels about the *third* is 188, and about the *fourth* it is 476. But 476 is equal to 188 plus 36 multiplied by 8; and similarly for the moment with reference to any other point. (See Du Bois's Strains in Framed Structures.)

This diagram may be used for finding *reactions, shears,* and *moments.* Thus

Let $l =$ the length of the truss, and the other notation as in Art. 46. Then, from (1) of Art. 46, the left reaction is

$$R = \frac{1}{l}\,(Pd + Px + \tfrac{1}{2}\,wx^2),$$

which for the passenger locomotive is

$$R = \frac{1}{l}\left(2388 + 88\,x + \frac{1}{2}wx^2\right) \quad . \quad . \quad . \quad . \quad (1)$$

and for the decapod engine is

$$R = \frac{1}{l}\left(2910 + 112\,x + \frac{1}{2}\,wx^2\right) . \quad . \quad . \quad . \quad (2)$$

Suppose, for example, that the third tender wheel of the decapod engine is 2 feet to the left of the right abutment. We take out the numbers 2181.8 and 102 from the diagram, and the reaction is

$$R=\frac{1}{l}(2181.8+102\overset{\times}{\cancel{+}}2).$$

In practice it is convenient to draw a skeleton outline of the truss to the same scale as the diagram, to be placed directly above it in the proper position for the maximum stress in each member.

Prob. 127. If the span be 120 feet, find the reactions for the passenger locomotive, (1) when the last tender wheel is 5 feet to the left of the right abutment, (2) when the first driver is 50 feet to the right of the left abutment, and (3) when the second pilot wheel is 40 feet to the left of the right abutment.

Ans. (1) 21.4 tons; (2) 52.1 tons; (3) 16.4 tons.

Prob. 128. If the span be 100 feet, find the moments for the decapod engine, (1) at an apex 30 feet to the left of the right abutment when the second driver is at that apex, and (2) at the center of the span when the first driver is there.

Ans. (1) 1341.4 foot-tons; (2) 1883 foot-tons.

Prob. 129. A through Pratt truss, Fig. 60, has 8 panels, each 18 feet long and 24 feet deep: find the maximum chord stresses due to a passenger locomotive and tender followed by a uniform train load of 3000 lbs. per linear foot.

To find the position for the maximum moment in 2–4 caused by the live load, we try the first driver at the panel point 4. Then for the uniform train load we have $x=7\times 18-36=90$ feet. In this position, the total live load on the truss

$$=88+1.5\times 90=223 \text{ tons}.$$

Since $8 + 8 < \frac{1}{8} \times 223$, and $8 + 8 + 20 > \frac{1}{8} \times 223$, this is the correct position for maximum moment in 2–4.

For this load the reaction, by formula (1), is

$$R = \tfrac{1}{144}(2388 + 88 \times 90 + .75 \times \overline{90}^2) = 113.8 \text{ tons};$$

and the moment at 3 is

$$M = 113.8 \times 18 - 188 = 1860.4.$$

$$\therefore \text{ max. stress in } 2\text{–}4 = \frac{1860.4}{24} = 77.5 \text{ tons} = \text{stress in } 4\text{–}6.$$

Similarly, the maximum stresses in the other three chord members are found to be the following:

$$\text{Max. stress in } 3\text{–}5 = -128.3, \text{ in } 5\text{–}7 = -157.0,$$
$$\text{in } 7\text{–}9 = -165.8 \text{ tons}.$$

NOTE.—To determine the maximum stress in 7–9 put 10 feet of the train to the left of the point 10; that is, put the third tender wheel at 8. Then

$$W = 88 + 1.5 \times 82 = 211 \text{ tons}.$$

But $88 + 10 \times 1.5 < \frac{1}{2} \times 211$; therefore this is *about* the correct loading.

$$R = \tfrac{1}{144}(2388 + 88 \times 82 + .75 \times \overline{82}^2) = 101.7 \text{ tons}.$$

$$M = 101.7 \times 72 - (2388 + 88 \times 10 + .75 \times \overline{10}^2)$$
$$= 3979.4 \text{ foot-tons}.$$

$$\therefore \text{ max. stress in } 7\text{–}9 = -\frac{3979.4}{24} = -165.8 \text{ tons}.$$

Prob. 130. A through Pratt truss, Fig. 60, has 8 panels, each 20 feet long and 24 feet deep: find (1) the maximum chord stresses, and (2) the maximum web stresses, in each panel, due to a decapod engine and tender followed by a uniform train load of 3000 lbs. per linear foot.

(1) To find the position for maximum stress in 2–4 we try the 2d driver at 4; then $x = 102.6$ feet; and

$$W = 112 + 1.5 \times 102.6 = 266 \text{ tons.}$$

Since $\frac{1}{8} \times 266 = 33\frac{1}{4} > 8 + 12.8$, and $< 8 + 12.8 + 12.8$, this is the correct position for maximum stress in 2–4, although if the *third* driver should be put at 4 we would get about the same stress, as $8 + 12.8 + 12.8$ would be just less than $\frac{1}{8}$ of the load on the truss.* The reaction is

$$R = \tfrac{1}{160}\,(2909.9 + 112 \times 102.6 + .75 \times \overline{102.6}^2) = 139.3 \text{ tons.}$$

$$\therefore \text{ max. stress in } 2\text{–}4 = \frac{139.3 \times 20 - 152.4}{24} = 109.7 \text{ tons}$$

$$= \text{stress in } 4\text{–}6.$$

Similarly, the maximum stresses in the other three chord members are found to be -181.5, -218.9, and -226.4 tons.

(2) To find the position for maximum shear in 2–3 put the 3d driver at 4; then $x = 106.8$ feet, and

$$W = 112 + 1.5 \times 106.8 = 272.2 \text{ tons.}$$

Since $8 + 12.8 + 12.8 < \frac{1}{8} \times 272.2$, this is the correct position for maximum shear in 2–3. The reaction is

$$R = \frac{1}{160}\,(2909.9 + 112 \times 106.8 + .75 \times \overline{106.8}^2) = 146.4 \text{ tons};$$

and shear in

$$2\text{–}4 = 146.4 - \frac{12.8}{20}\,(4.25 + 8.5) - \frac{8}{20} \times 16.5 = 131.64 \text{ tons.}$$

$$\therefore \text{ max. stress in } 2\text{–}3 = -131.64 \times 1.302 = -171.4 \text{ tons.}$$

* The conditions of Arts. 47 and 48 may sometimes be satisfied by different positions of the load.

If the 2d driver be put at 4 we shall obtain about the same result.

The following stresses are found in a manner similar to the above:

Maximum Stresses in the *Diagonals*.	Maximum Stresses in the *Verticals*.
2–3 = – 171.4 tons.	3–4 = + 51.1 tons.
3–6 = + 130.7 tons.	5–6 = – 72.7 tons.
5–8 = + 94.7 tons.	7–8 = – 48.9 tons.
7–10 = + 63.6 tons.	9–10 = – 29.4 tons.
9–8′ = + 38.3 tons.	
7′–6′ = + 18.0 tons.	

Prob. 131. A through Pratt truss has 7 panels, each 20 feet long and 20 feet deep; the dead load is 1600 lbs. per linear foot, the live load is a passenger locomotive and tender followed by a uniform train load of 3000 lbs. per linear foot: find the maximum stresses in all the members.

CHORD STRESSES.

MEMBERS.	2–4, 4–6	3–5	5–7	7–7′
Dead load stresses	+ 48.0	– 80.0	– 96.0	– 96.0
Live load stresses	+ 98.3	–157.3	–185.4	–185.4
Maximum stresses	+146.3	–237.3	–281.4	–281.4

STRESSES IN THE DIAGONALS.

MEMBERS.	2–3	3–6	5–8	7–8′	7–6′
Dead load stresses .	– 67.9	+ 45.2	+22.6	0.0	0.0
Live load stresses .	–138.9	+ 98.7	+64.6	+36.5	+16.7
Maximum stresses .	–206.8	+143.9	+87.2	+36.5	– 5.9

The live load stress in $7'–6'$ is $+16.7$, while the dead load stress in $5'–8'$ or 5–8 is $+22.6$. Since the member $7'–6'$ cannot take compression, this result shows that the counter $7'–6'$ is not needed for this loading, the member $5'–8'$ taking the shear as tension.

STRESSES IN THE VERTICALS.

MEMBERS.	3–4	5–6	7–8
Dead load stresses	+16.0	−16.0	− 0.0
Live load stresses	+39.6	−45.7	−25.8
Maximum stresses	+55.6	−61.7	−25.8

Prob. 132. A through Pratt truss, Fig. 33, has 10 panels, each 20 feet long and 20 feet deep; the dead load is 2000 lbs. per foot, the live load is a decapod engine and tender followed by a uniform train load of 3000 lbs. per foot: find the maximum stresses in all the members.

CHORD STRESSES.

MEMBERS.	2–4, 4–6	6–8	8–10	10–12	9–11
Dead load stresses .	+ 90.0	+160.0	+210.0	+240.0	−250.0
Live load stresses .	+163.0	+280.6	+357.7	+398.0	−406.6
Maximum stresses .	+253.0	+440.6	+567.7	+638.0	−656.6

STRESSES IN THE DIAGONALS.

MEMBERS.	2–3	3–6	5–8	7–10	9–12	11–10′	9′–8′
Dead load stresses . . .	−127.3	+ 98.9	+ 70.7	+ 42.4	+14.1	0.0	0.0
Live load stresses	−230.5	+186.5	+146.9	+111.4	+80.2	+54.1	+33.1
Maximum stresses	−357.8	+285.4	+217.6	+153.8	+94.3	+40.0	− 9.3

STRESSES IN THE VERTICALS.

MEMBERS.	3–4	5–6	7–8	9–10	11–12
Dead load stresses . . .	+20.0	− 50.0	− 30.0	−10.0	− 0.0
Live load stresses . . .	+51.1	−103.9	− 78.8	−56.7	−38.3
Maximum stresses . . .	+71.1	−153.9	−108.8	−66.7	−38.3

Since the live load stress in 11–10′ or 11–10 is + 54.1, while the dead load stress in the same diagonals is − 14.1, the maximum stress is the difference, or + 40.0. Also, since the live load stress in 9′–8′ or 9–8 is + 33.1, while the dead load stress is − 42.4, the maximum stress in 9′–8′ or 9–8 is − 9.3. Hence counters are needed only in the two middle panels.

Prob. **133.** A through Warren truss has 8 panels, each 15 feet long and 15 feet deep; the dead load is 2000 lbs. per linear foot, the live load is a passenger locomotive and tender followed by a uniform train load of 3000 lbs. per linear foot: find the maximum stresses in all the members.

Prob. **134.** A deck Pratt truss has 10 panels, each 20 feet long and 24 feet deep; the dead load is given by formula (1), Art. 15, the live load is a decapod engine and tender followed by a uniform train load of 3000 lbs. per linear foot: find the maximum stresses in all the members.

CHAPTER IV.

MISCELLANEOUS TRUSSES.

Roof Trusses.

Art. 50. The King and Queen Truss — The Fink Truss. — The types of roof trusses most commonly used for spans from 30 feet to 100 feet, and even to 130 feet, are shown in Chapter I. The King and Queen truss, Fig. 7, is a common form for *wooden trusses*, or for trusses that are all of wood except the verticals, which are iron or steel tie-rods. This type of truss is sometimes used also when it is entirely made of steel.

The *Belgian*, or *Fink* roof-truss, Fig. 10, is a very common and economical type of truss for spans up to 130 feet. The struts, 3–4, 5–6, 7–8, are normal to the rafter, dividing it into equal parts, and, with the upper chord, are made either of wood or iron. The other members are ties, and are made of iron or steel. This type of truss is often entirely of steel, and is commonly used for iron or steel roofs over mills, shops, warehouses, train-sheds, etc. Some of the largest trusses of this type are those in the car-shops of the Pennsylvania railroad at Altoona, Pa., having a span of 132 feet.

The slope of the rafter is usually determined by the kind of roof covering used. Slate should not be used on a roof when the slope is less than 1 vertical to 3 horizontal, and preferably 1 vertical to 2 horizontal. Gravel should not be used on a slope greater than 1 vertical to 4 horizontal. Tin

may be used on any slope, or on a flat roof. Corrugated iron should not be used on a slope less than 1 to 3; for flatter roofs than 1 to 3, of corrugated iron, are liable to leak under a driving rain as the usual joints are not tight. When possible the slope should not be less than 1 to 2.

The following table gives the approximate weight per square foot of roof coverings, exclusive of steel construction.

APPROXIMATE WEIGHT PER SQUARE FOOT OF ROOF COVERINGS.

Corrugated iron, unboarded, No. 26 to No. 18	1 to 3 lbs.
Felt and asphalt, without sheathing	2 lbs.
Felt and gravel, without sheathing	8 to 10 lbs.
Slate, without sheathing, $\frac{3}{16}''$ to $\frac{1}{4}''$	7 to 9 lbs.
Copper, without sheathing	1 to $1\frac{1}{2}$ lbs.
Tin, without sheathing	1 to $1\frac{1}{2}$ lbs.
Shingles, with lath	$2\frac{1}{4}$ lbs.
Skylight of glass, $\frac{3}{16}''$ to $\frac{1}{2}''$, including frame	4 to 10 lbs.
White pine sheathing, 1″ thick	3 lbs.
Yellow pine sheathing, 1″ thick	4 lbs.
Spruce sheathing, 1″ thick	2 lbs.
Lath and plaster ceiling	8 to 10 lbs.
Tile, flat	15 to 20 lbs.
Tile, corrugated	8 to 10 lbs.
Tile, on 3″ fireproof blocks	30 to 35 lbs.

The weight of the steel roof construction must be added to the above. For ordinary light roofs without ceilings, the weight of the steel construction may be taken at 5 lbs. per square foot for spans up to 50 feet, and 1 lb. additional for each 10 feet increase of span. (Manual of useful information and tables appertaining to the use of Structural Steel, as manufactured by the Passaic Rolling Mill Co., Paterson, N.J. By George H. Blakeley, C.E.)

Prob. 135. A truss, Fig. 62, with one end free, has its span 120 feet, its rise 30 feet, the ties, 3–4, 5–6, 7–8, 9–10,

11–12, vertical, dividing the rafter into 6 equal parts; the dead load per panel is 2.5 tons, snow load per panel 1.5 tons, normal wind load per panel, wind on fixed side, 2 tons: find

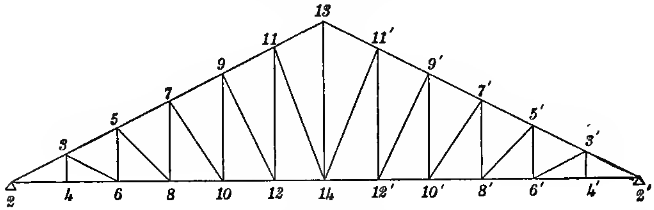


Fig. 62

all the stresses in all the members of the half of the truss on the windward side.

STRESSES IN THE LOWER CHORD.

MEMBERS.	2–6	6–8	8–10	10–12	12–14
Dead load stresses . . .	+27.5	+25.0	+22.5	+20.0	+17.5
Snow load stresses . . .	+16.5	+15.0	+13.5	+12.0	+10.5
Wind load stresses . . .	+17.8	+15.6	+13.4	+11.1	+ 8.9
Maximum stresses . . .	+61.8	+55.6	+49.4	+43.1	+36.9

STRESSES IN THE UPPER CHORD.

MEMBERS.	2–3	3–5	5–7	7–9	9–11	11–13
Dead load stresses	–30.75	–27.95	–25.15	–22.35	–19.55	–16.75
Snow load stresses	–18.45	–16.77	–15.09	–13.41	–11.73	–10.05
Wind load stresses	–14.52	–13.00	–11.48	– 9.96	– 8.44	– 7.44
Maximum stresses	–63.72	–57.72	–51.72	–45.72	–39.72	–34.24

STRESSES IN THE VERTICALS.

MEMBERS.	5–6	7–8	9–10	11–12	13–14
Dead load stresses . .	+1.25	+2.50	+3.75	+ 5.00	+12.50
Snow load stresses . .	+0.75	+1.50	+2.25	+ 3.00	+ 7.50
Wind load stresses .	+1.12	+2.24	+3.36	+ 4.46	+ 5.56
Maximum stresses . .	+3.12	+6.24	+9.36	+12.46	+25.56

(3–4 is not necessary to the stability of the truss.)

STRESSES IN THE DIAGONALS.

MEMBERS.	3–6	5–8	7–10	9–12	11–14
Dead load stresses .	–2.80	–3.52	– 4.50	– 5.55	– 6.70
Snow load stresses .	–1.68	– 2.11	– 2.70	– 3.33	– 4.02
Wind load stresses .	–2.48	–3.16	– 4.04	– 5.00	– 6.00
Maximum stresses .	–6.96	–8.79	–11.24	–13.88	–16.72

Prob. **136.** A truss of the type of Fig. 62, with one end free, has its span 150 feet, its rise 25 feet, the ties, 3–4, 5–6, etc., vertical, dividing the rafter into six equal parts; the dead load per panel is 2.8 tons, snow load per panel 1.5 tons, normal wind load per panel, wind on fixed side, 1.8 tons: find all the stresses in all the members of the half of the truss on the windward side.

STRESSES IN THE LOWER CHORD.

MEMBERS.	2–6	6–8	8–10	10–12	12–14
Dead load stresses . .	+46.20	+42.00	+37.80	+33.60	+29.40
Snow load stresses . .	+24.75	+22.50	+20.25	+18.00	+15.75
Wind load stresses . .	+22.79	+19.93	+17.06	+14.20	+11.34
Maximum stresses . .	+93.74	+84.43	+75.11	+65.80	+56.49

STRESSES IN THE UPPER CHORD.

MEMBERS.	2–3	3–5	5–7	7–9	9–11	11–13
Dead load stresses .	−48.97	−44.52	−40.07	−35.62	−31.16	−26.71
Snow load stresses .	−26.24	−23.85	−21.47	−19.08	−16.70	−14.31
Wind load stresses .	−20.74	−18.32	−15.91	−13.50	−11.09	− 8.96
Maximum stresses .	−95.95	−86.69	−77.45	−68.20	−58.95	−49.98

STRESSES IN THE VERTICALS.

MEMBERS.	5–6	7–8	9–10	11–12	13–14
Dead load stresses . . .	+1.40	+2.80	+4.20	+ 5.60	+17.02
Snow load stresses . .	+0.75	+1.50	+2.25	+ 3.00	+ 9.12
Wind load stresses . . .	+0.94	+1.89	+2.84	+ 3.78	+ 5.76
Maximum stresses . . .	+3.09	+6.19	+9.29	+12.38	+31.90

STRESSES IN THE DIAGONALS.

MEMBERS.	3–6	5–8	7–10	9–12	11–14
Dead load stresses . . .	− 5.15	− 5.60	− 6.27	− 7.28	− 8.34
Snow load stresses. . .	− 2.76	− 3.00	− 3.36	− 3.90	− 4.47
Wind load stresses . . .	− 3.38	− 3.82	− 4.27	− 4.97	− 5.65
Maximum stresses . . .	−11.29	−12.42	−13.90	−16.15	−18.46

Prob. **137.** A Fink truss, Fig. 10, with one end free, has its span 150 feet, its rise 25 feet, the rafter divided into four equal parts by struts drawn normal to it, the dead load per panel 3 tons, the snow load per panel 1.5 tons, the normal wind load per panel, wind on fixed side, 2 tons: find all the stresses in all the members of the half of the truss on the windward side.

STRESSES IN THE LOWER CHORD.

MEMBERS.	2–4	4–6	6–10
Dead load stresses	+31.50	+27.00	+18.00
Snow load stresses	+15.75	+13.50	+ 9.00
Wind load stresses	+15.84	+12.68	+ 6.34
Maximum stresses	+63.09	+53.18	+33.34

STRESSES IN THE UPPER CHORD.

MEMBERS.	2–3	3–5	5–7	7–9
Dead load stresses	−33.15	−32.22	−31.29	−30.36
Snow load stresses	−16.58	−16.11	−15.65	−15.18
Wind load stresses	−14.36	−14.36	−14.36	−14.36
Maximum stresses	−64.09	−62.69	−61.30	−59.90

STRESSES IN THE WEB MEMBERS.

MEMBERS.	3–4, 7–8	5–6	4–5, 5–8	6–8	8–9
Dead load stresses . . .	−2.85	− 5.70	+4.50	+ 9.00	+13.50
Snow load stresses . . .	−1.43	− 2.85	+2.25	+ 4.50	+ 6.75
Wind load stresses. . .	−2.00	− 4.00	+3.12	+ 6.32	+ 9.48
Maximum stresses . .	−6.28	−12.55	+9.87	+19.82	+29.73

Art. 51. The Crescent Truss, Figs. 63 and 64, is a good form for comparatively large spans. Riveted iron-work is used throughout.

Prob. 138. A circular Crescent truss, Fig. 63, with one end free, has its span 160 feet, the rise of the upper chord 32 feet, the rise of the lower chord 20 feet, the number of

panels 8, the apexes of both the upper and lower chords lying on arcs of circles, dividing them into 8 equal parts; the dead load is 3 tons per panel, the wind load is 2 tons

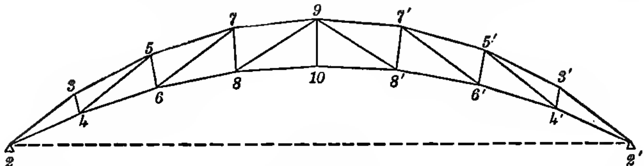


Fig. 63

per panel, the wind being supposed to act vertically on the fixed side only:[1] find all the stresses in all the members of the half of the truss on the windward side.

The computation of the stresses is effected by the application of the principles of Chapter I.

The radius of the upper chord = 116 feet.

The radius of the lower chord = 170 feet.

The lengths of 2–3, 3–5, etc. = 22.04 feet.

The lengths of 2–4, 4–6, etc. = 20.81 feet.

The horizontal distance of each apex from the left abutment 2, and its vertical distance above the line 2–2′, are as follows:

	(3)	(5)	(7)	(9)
Hor. distance from 2,	17.33,	36.94,	58.06,	80 feet.
Height above line 2–2′,	13.61,	23.70,	29.91,	32 feet.

	(4)	(6)	(8)	(10)
Hor. distance from 2,	18.93,	38.76,	59.22,	80 feet.
Height above line 2–2′,	8.65,	14.92,	18.72,	20 feet.

[1] If the normal wind pressure were taken, it would have a different value for each panel, owing to the curved surface of the rafters, which would increase the difficulty of the computation.

The dead load reaction = 10.5 tons.

The stresses in the chords are best found by moments; for the other members the method of resolution of forces may be used.

Thus, to compute the dead load stress in 2–3, we find the lever arm of 2–3 to be 4.89 feet.

$$\therefore \text{ dead load stress in } 2\text{–}3 = -\frac{10.5 \times 18.93}{4.89} = -40.7 \text{ tons.}$$

$$\text{Similarly, the stress in } 2\text{–}4 = \frac{10.5 \times 17.33}{5.18} = 35.1 \text{ tons.}$$

To find the stresses in 3–4 and 3–5 pass a section cutting 2–4, 3–4, and 3–5. Then, denoting the stresses in 3–4 and 3–5 by s_1 and s_2, we have, for horizontal and vertical components respectively,

$$35.1 \times .9095 + \frac{1.6}{\sqrt{1.6^2 + 4.96^2}} s_1 + .889\, s_2 = 0,$$

and
$$35.1 \times .4156 - \frac{4.96}{5.21} s_1 + .4579\, s_2 + 7.5 = 0.$$

$$\therefore\ s_1 = +\ 5.0 \text{ tons} = \text{stress in } 3\text{–}4;$$

and
$$s_2 = -\ 37.7 \text{ tons} = \text{stress in } 3\text{–}5.$$

The wind load reaction = 5.1 tons.

$$\therefore \text{ wind load stress in } 2\text{–}3 = -\frac{5.1 \times 18.93}{4.89} = -19.7 \text{ tons.}$$

In this way all the stresses may be found.

STRESSES IN THE TOP CHORD.

MEMBERS.	2–3	3–5	5–7	7–9
Dead load stresses . . .	−40.7	−37.7	−37.8	−37.8
Wind load stresses . . .	−19.7	−17.0	−16.6	−14.8
Maximum stresses . . .	−60.4	−54.7	−54.4	−52.6

STRESSES IN THE BOTTOM CHORD.

MEMBERS.	2–4	4–6	6–8	8–10
Dead load stresses . . .	+35.1	+36.9	+37.8	+38.1
Wind load stresses . . .	+16.0	+16.4	+15.0	+12.7
Maximum stresses . . .	+51.1	+53.3	+52.8	+50.8

STRESSES IN THE WEB MEMBERS.

MEMBERS.	3–4	5–6	7–8	9–10	4–5	6–7	8–9
Dead load stresses	+5.0	−1.8	+5.0	−1.4	+5.1	−0.6	+4.2
Wind load stresses	+2.2	+2.0	+2.8	+1.4	−2.0	−2.0	+2.4
Max. compression	+0.0	−1.8	+0.0	−1.4	+0.0	−2.6	+0.0
Max. tension . . .	+7.2	+0.2	+7.8	+0.0	+5.1	−0.0	+6.6

Prob. **139.** A circular Crescent truss, Fig. 64, with one end free, has its span 160 feet, the rise of the upper chord

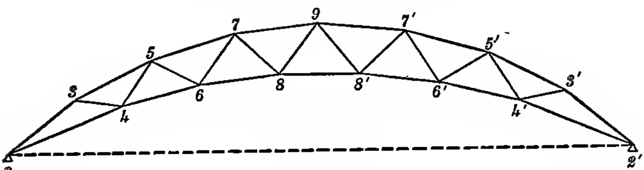


Fig. 64

32 feet, the rise of the lower chord 20 feet, the apexes of both the upper and lower chords lying on arcs of circles. The upper chord has 8 panels, all of the same length. The lower chord has 7 panels; each of the 5 panels 4–6, 6–8,

etc., is equal to one-eighth of the lower chord, while the two end panels, 2–4 and 2′–4′, are each equal to $1\frac{1}{2}$ of the other panels, that is, equal to $1\frac{1}{2}$ eighths of the lower chord; the dead load is 3 tons per panel and the wind load is 2 tons per panel, the wind being supposed to act vertically on the fixed side: find all the stresses in all the members of the half of the truss on the windward side.

Here the radii of the upper and lower chords are 116 and 170 feet, respectively, as in Prob. 138, and the coördinates of the apexes of the upper chord are also the same as in Prob. 138.

The length of 2–4 is found to be 31.21 feet, and the coördinates of 4 to be 28.77 feet and 12.09 feet.

The reactions are the same as in Prob. 138.

$$\therefore \text{Dead load stress in } 2\text{–}3 = -\frac{10.5 \times 28.77}{8.26} = -36.6 \text{ tons.}$$

$$\text{Dead load stress in } 2\text{–}4 = \frac{10.5 \times 17.33}{5.83} = 31.2 \text{ tons.}$$

$$\text{Wind load stress in } 2\text{–}3 = -\frac{5.1 \times 28.77}{8.26} = -17.7 \text{ tons.}$$

$$\text{Wind load stress in } 2\text{–}4 = \frac{5.1 \times 17.33}{5.83} = 15.1 \text{ tons.}$$

STRESSES IN THE TOP CHORD.

MEMBERS.	2–3	3–5	5–7	7–9
Dead load stresses	−36.6	−40.5	−39.3	−38.7
Wind load stresses	−17.7	−18.4	−16.8	−14.4
Maximum stresses	−54.3	−58.9	−56.1	−53.1

Stresses in the Bottom Chord.

Members.	2–4	4–6	6–8	8–8′
Dead load stresses	+31.2	+35.1	+36.6	+36.6
Wind load stresses.	+15.1	+16.2	+14.7	+12.4
Maximum stresses	+46.3	+51.3	+51.3	+49.0

Stresses in the Web Members.

Members.	3–4	4–5	5–6	6–7	7–8	8–9
Dead load stresses . .	+ 7.2	+3.2	+4.1	+2.7	+3.1	+3.0
Wind load stresses . .	+ 3.0	+1.4	+2.2	+2.6	+2.9	+2.7
Maximum stresses. . .	+10.2	+4.6	+6.3	+5.3	+6.0	+5.7

Bridge Trusses.

Art. 52. The Pegram Truss — The Parabolic Bow-string Truss. — The Pegram truss, Fig. 65, consists of the same number of panels in each chord. All the panels of

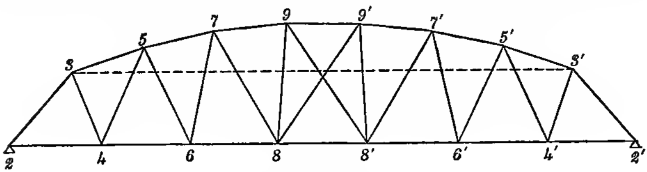


Fig. 65

each chord are of equal length, the upper chord panels being shorter than the lower. The apexes of the upper chord lie on an arc of a circle, the chord of which, 3–3′, is about one and one half panel lengths shorter than the span. The rise of the upper chord is such as to make the posts,

4–5, 6–7, 8–9, nearly equal in length, or it may be so taken that the posts will decrease in length toward the ends. In a deck bridge the upper chord is made straight and the lower chord curved.

Prob. 140. A through Pegram truss, Fig. 65, has a span of 200 feet, divided into 7 panels, each 28.57 feet long; the upper apexes lie on an arc of a circle, the center height being 15 feet above the chord 3–3′, which is 160 feet long; the upper panels are all of equal length, the members 2–3, 4–5, 6–7, 8–9 are struts, all the remaining web-members are ties, the dead and live loads are 10 tons and 18 tons per panel per truss: find the stresses in all the members.

Here the radius of the upper chord = 220.83 feet.

The lengths of the upper panels 3–5, 5–7, etc. = 23.37 feet.

The horizontal distance of each upper apex from the left abutment 2, and its vertical height above the lower chord 2–2′, are the following:

	(3)	(5)	(7)	(9)
Hor. distance from 2,	20.00,	42.20,	65.05,	88.05.
Height above lower chord,	24.00,	31.29,	36.23,	38.65.

The dead load reaction = 30 tons.

Then, dead load stress in

$$4\text{–}6 = \frac{30 \times 42.20 - 10 \times 13.63}{31.29} = 36.1 \text{ tons.}$$

Also, dead load stress in 9–9′

$$= -\frac{30 \times 4 \times 28.57 - 60 \times 28.57}{38.65} = -44.3 \text{ tons.}$$

Similarly, dead load stress in 3–5

$$= -\frac{30 \times 28.57}{25.47} = -33.6 \text{ tons.}$$

To find the dead load stress in any web member, as 5–6, pass a section cutting 5–7, 5–6, and 4–6, and take moments around the intersection of 5–7 and 4–6, which is 102.53 feet to the left of 2.

$\therefore$ dead load stress in 5–6

$$= \frac{30 \times 102.53 - 10 \times 131.1}{144.1} = 12.2 \text{ tons.}$$

The stresses in the other web members are found in like manner.

The live load stresses in the chords and in 2–3 and 3–4 are a maximum for a full load, and may therefore be found by multiplying the corresponding dead load stresses by 1.8.

For a maximum in 5–6 and 4–5 the live load covers all the joints from the right to 6. The reaction for this loading is

$$R = \frac{18}{7}(1+2+3+4+5) = \frac{18 \times 15}{7} \text{ tons.}$$

$$\therefore \text{ live load stress in 5–6} = \frac{18 \times 15}{7} \times \frac{102.53}{144.1} = 27.4 \text{ tons.}$$

For a maximum in 9–8′ the joints 4′, 6′, and 8′ are loaded. The reaction for this loading is

$$R = \tfrac{18}{7}(1+2+3) = \tfrac{108}{7} \text{ tons,}$$

which is also the maximum shear in this panel. Since the upper and lower chord members of this panel are horizontal, the stress in 9–8′ is found by Art. 18.

$$\therefore \text{ max. stress in 9–8}' = \frac{108}{7} \times \frac{46.41}{38.65} = 18.5 \text{ tons.}$$

STRESSES IN THE UPPER CHORD.

MEMBERS.	3–5	5–7	7–9	9–9′
Dead load stresses	−33.6	− 42.4	− 45.1	− 44.3
Live load stresses	−60.4	− 76.2	− 81.1	− 79.7
Maximum stresses	−94.0	−118.6	−126.2	−124.0

STRESSES IN THE LOWER CHORD.

MEMBERS.	2–4	4–6	6–8	8–8′
Dead load stresses	+25.0	+ 36.1	+ 41.6	+ 44.6
Live load stresses	+45.0	+ 65.0	+ 74.9	+ 80.3
Maximum stresses	+70.0	+101.1	+116.5	+124.9

STRESSES IN THE WEB TIES.

MEMBERS.	3–4	5–6	7–8	9–8′	9′–6′
Dead load stresses . .	+20.6	+12.2	+ 6.2	+ 0.0	+ 0.0
Live load stresses . .	+37.2	+27.4	+22.7	+18.5	+14.1
Maximum stresses . .	+57.8	+39.6	+28.9	+18.5	+14.1

STRESSES IN THE WEB STRUTS.

MEMBERS.	2–3	4–5	6–7	8–9
Dead load stresses . .	− 38.8	−10.6	− 1.3	− 0.0
Live load stresses . . .	− 69.8	−27.8	−17.3	−12.0
Maximum stresses . .	−108.6	−38.4	−18.6	−12.0

See Framed Structures, by Johnson, Bryan, and Turneaure; also Engineering News, Dec. 10 and 17, 1887, and Feb. 14, 1891, where the answers differ very slightly from the above on account of the center member 9–9′ being $\frac{1}{20}$ less.

Prob. **141.** A through parabolic bowstring truss, Fig. 66, has 8 panels, each 24 feet long, and 32 feet center depth,

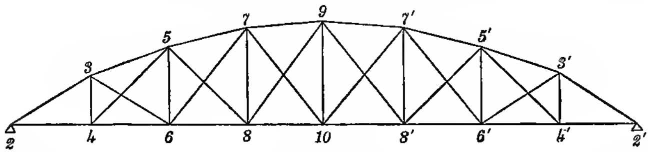


Fig. 66

the verticals are ties and the diagonals are struts; the dead and live loads are 10 tons and 15 tons per panel per truss: find the stresses in all the members.

Stresses in the Top Chord.

Members.	2–3	3–5	5–7	7–9
Dead load stresses	− 69.4	− 65.0	− 61.8	− 60.2
Live load stresses	−104.1	− 97.5	− 92.7	− 90.3
Maximum stresses	−173.5	−162.5	−154.5	−150.5

Dead load stress in each panel of lower chord = + 60 tons.
Live load stress in each panel of lower chord = + 90 tons.
Maximum stress in each panel of lower chord = + 150 tons.

Stresses in the Diagonals.

Members.	3–6	5–8	7–10	4–5	6–7	8–9
Dead load stresses .	0.0	0.0	0.0	0.0	0.0	0.0
Live load stresses .	−13.0	−15.9	−18.0	−15.9	−18.0	−18.1
Maximum stresses .	−13.0	−15.9	−18.0	−15.9	−18.0	−18.1

STRESSES IN THE VERTICALS.

MEMBERS.	3–4	5–6	7–8	9–10
Dead load stresses	+10.0	+10.0	+10.0	+10.0
Live load stresses	+15.0	+19.7	+22.5	+23.4
Maximum stresses	+25.0	+29.7	+32.5	+33.4

Art. 53. Skew Bridges are those in which the end supports of one truss are not directly opposite to those of the other. The trusses of a skew bridge are usually placed so that the intermediate panel points are directly opposite in the two trusses, and the floor beams are at right angles to the trusses. When the skew is the same at each end the trusses are *symmetrical;* otherwise they are *unsymmetrical.* In the analysis of unsymmetrical trusses, each truss must be treated separately; and the stresses are to be computed for all the members of the truss.

Prob. 142. Fig. 68 is a plan and Fig. 67 is the elevation of one of the two trusses of an unsymmetrical through

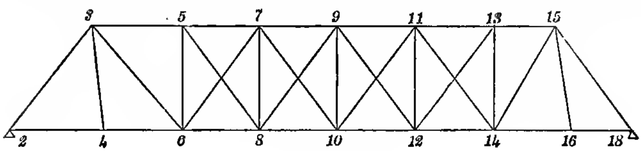


Fig. 67

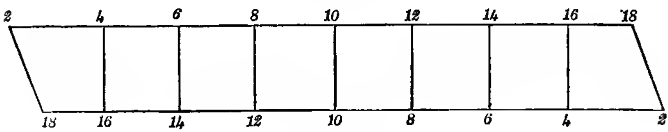


Fig. 68

Pratt bridge; the span 2–18 is 120 feet, the depth is 20 feet, the panels 3–5 and 2–4 are each 18 feet, the panels 13–15 and 16–18 are each 12 feet, the other panels are each 15 feet, the inclination of the end posts 2–3 and 15–18 is the same as that of the diagonals 5–8, 7–10, 10–11, 12–13, etc., and the inclination of the two hip verticals 3–4 and 15–16 is the same; the dead and live loads are 1000 lbs. and 2000 lbs. per foot per truss: find all the stresses in all the members.

We first find the chord stresses due to the dead load as follows:

Dead panel load at $4 = \frac{1}{2}(9 + 7.5) = 8.25$ tons.

Dead panel load at $16 = \frac{1}{2}(6 + 7.5) = 6.75$ tons.

Dead panel load at 6, 8, 10, 12, $14 = 7.50$ tons.

Dead load reaction at $2 = 25.5$ tons.

Dead load reaction at $18 = 27.0$ tons.

tan $\angle$ 3–2 with vertical $= .75 =$ tan $\angle$ 15–18 with vertical.

tan $\angle$ 3–4 with vertical $= .15 =$ tan $\angle$ 15–16 with vertical.

tan $\angle$ 3–6 with vertical $= .9$.

tan $\angle$ 14–15 with vertical $= .6$.

Hence by (2) of Art. 19 we have the following dead load stresses:

Dead load stress in 2–4 $= 25.5 \times .75 = 19.1$ tons.

Dead load stress in 4–6 $= 19.1 + 8.25 \times .15 = 20.3$ tons.

Dead load stress in 16–18 $= 27.0 \times .75 = 20.3$ tons.

Dead load stress in 14–16 $= 20.3 - 6.75 \times .15 = 19.3$ tons.

etc. etc. etc.

Also the following greatest live load stresses:

$$\text{Stress in } 3\text{–}6 = \tfrac{1}{120}[13.5 \times 12 + 15 \times 285]\,1.345$$
$$= +\,49.7 \text{ tons.}$$

$$\text{Stress in } \quad 5\text{–}6 = -\tfrac{1}{120}[162 + 15 \times 198] = -\,26.1 \text{ tons.}$$

$$\text{Stress in } 12\text{–}13 = \tfrac{1}{120}[297 + 15 \times 222]\,1.166 = +\,35.2 \text{ tons.}$$

CHORD STRESSES.

MEMBERS.	2–4	4–6	6–8	8–10	10–12	12–14	14–16	16–18	7–11
Dead load stresses . . .	+19.1	+20.8	+ 35.8	+ 48.1	+ 41.0	+31.5	+19.3	+20.3	− 44.8
Live load stresses . . .	+38.2	+40.6	+ 71.6	+ 86.2	+ 82.0	+63.0	+38.6	+40.6	− 89.6
Maximum stresses . . .	+57.3	+60.9	+107.4	+129.3	+123.0	+94.5	+57.9	+60.9	−134.4

STRESSES IN THE DIAGONALS.

MEMBERS.	3–4	3–6	5–8	7–10	9–12	11–14
Dead load stresses.	+ 8.3	+23.2	+12.2	+ 2.8	− 6.6	−15.9
Live load stresses .	+16.6	+49.7	+35.1	+23.0	+13.5	+ 6.3
Maximum stresses	+24.9	+72.9	+47.3	+25.8	+ 6.9	− 9.6

MEMBERS.	15–16	14–15	12–13	10–11	8–9	6–7
Dead load stresses.	+ 6.8	+23.6	+15.9	+ 6.6	− 2.8	−12.2
Live load stresses .	+13.6	+48.8	+35.2	+23.9	+14.7	+ 7.7
Maximum stresses.	+20.4	+72.4	+51.1	+30.5	+11.9	− 4.5

We see from the above that, theoretically, the diagonals 6–7 and 11–14 are not needed.

STRESSES IN THE POSTS.

MEMBERS.	2–3	5–6	7–8	9–10	11–12	11–14	15–18
Dead load stresses .	−31.9	− 9.8	− 2.3	− 0.0	− 5.3	−12.8	− 33.8
Live load stresses .	−63.8	−26.1	−17.1	−12.6	−20.5	−30.2	− 67.6
Maximum stresses .	−95.7	−35.9	−19.4	−12.6	−25.8	−43.0	−101.4

Prob. **143.** Let the dimensions of Fig. 67 be as follows: span = 144 feet, depth = 24 feet, the panels 3–5 and 2–4 = 21 feet, the panels 13–15 and 16–18 = 15 feet, all the other panels = 18 feet; let the dead and live loads be 800 lbs. and 2000 lbs. per foot per truss: find all the stresses in all the members.